Stochastic Optimal Control on a Biologically Inspired Robotic Finger

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Abstract—With the goal to build robotic hands which can reach the levels of dexterity and robustness of the hand, the question of what are the candidate control principles that can handle the nonlinearities, the high dimensionality and the internal noise of biomechanical structures of the complexity of the hand, is still open. In this work we present the first stochastic optimal feedback controller applied to a full tendon driven simulated robotic index finger. In our model we do take into account the full tendon structure of the index finger which consist of 11 tendons based on the underlying physiology and we consider muscle with the typical force - length and force velocity properties. Our feedback controller show robustness against noise and perturbation of the dynamics while it can also successfully handle the nonlinearities and high dimensionality of the robotic index finger. Furthermore as it is shown in the evaluations, it provides the complete time history of the tendon excursions and the tendon velocities of the index finger for the tasks of tapping with zero and nonzero terminal velocities.

I. INTRODUCTION

Despite the work on building “smart” and biologically inspired robotic hands there is no mechanical hand that can compete the robustness and the dexterity of the hand in tasks such as grasping objects with uncertain load and various shapes, writing skills, playing music instruments skills and manipulating objects. All these tasks involve, in different levels, motion and force control especially in cases where contact with surfaces occurs. The gap in the functionality and robustness between robotic and human hand has its origins in the lack of understanding the design principles of the biomechanical structure of the hand in strong relation with solid control theoretic frameworks.

From the control theoretic stand point, the control of a highly dimensional and nonlinear stochastic plant of the complexity of a robotic or bio-mechanical hand is not an easy task. The nonlinearities of the neuromuscular and/or Robotic plant and the large dimensionality of the state space make the understanding of the neuromuscular control of the hand a challenging problem. To appreciate the highly dimensionality it is enough to consider that for the control of the hand there 35 tendons controlled by the brain. Given the level of the complexity of the hand and in spite of the significant amount of work in the Biomechanics and Robotics community, it remains unknown

• What is the underlying control principles that could handle redundancy of the hand for even a simple finger movement given the fact that just only for the index finger there are 7 tendons that play the role of actuators.
• What is the benefit of having tendons as actuators? From the control theoretic point of view, the existence of the tendons increases the order of the system from a second order to third order system. What is the benefit of such a mechanical design?
• Under which principles the nervous system transitions between motion and force control in tasks where contact with an object occurs.

Our recent experimental work [8] investigated the neural control of contact transition between motion and force during tapping. In [6] we found that such transitions from motion to well-directed contact force are a fundamental part of dexterous manipulation, and that such tasks are controlled optimally. Moreover, one of the main assumption in [6] is that the underlying control strategy of the finger is considered to be open loop. In addition the tendon delays are modeled as activation contraction dynamics that drive the torques applied to 3 joints of the index finger. Even though with this simple model the optimality principles of the motion to force transition for the task of tapping were investigated, an open loop control strategy would have failed in tasks such as object manipulation where feedback control is critical requirement for successfully performing the manipulation task. Furthermore, since only 3 sets of differential equation that model the activation contraction dynamics are considered, the full structure and redundancy of the index finger is not explored.

Motivated by the limitations of previous work which has been mostly experimental, in this paper we are addressing the problem of controlling the index finger insight the framework of stochastic optimal feedback control theory. In particular, we make use of the iterative Linear Quadratic Gaussian controller (iLQG) - one of the few methods in optimal control that can handle nonlinear dynamical systems with complexity and dimensionality equivalent to the complexity and dimensionality of the index finger. Our index finger model is based on the accurate biomechanical model in [1] which consists of 11 tendons. Furthermore we are assuming the existence of muscles and therefore we make use of a muscle model [3] that encodes the typical force- velocity and force - length properties of muscles. Our stochastic optimal controller gives us the complete time history of the tendon excursions and tendon velocities for the tasks of tapping with nonzero and zero velocity at the point of contact and it provides the resulting torques at the 3 joints of the index finger.

In the rest of the paper, in section II we discuss the index finger model. In section III we derive the moment arm matrix of the index finger. In section IV we provide the basic equations our muscle model. In section V we discuss the iLQG framework and provide the main equations while in VI we provide the overall dynamic system which include the
multibody dynamics. In the experimental section we show our simulation results for the tapping task with index finger under non-zero and zero velocity at the contact point. Finally in the last section we conclude and discuss our future plans.

II. INDEX FINGER MODEL

The skeleton of the human index finger consist of 3 joints connected with 3 rigid links. The two joints the interphalangeal proximal (PIP) and the interphalangeal distal (DIP) are described as hinge joints that can generate both flexion - extension. The metacarpophalangeal joint (MCP) is a saddle joint and it can generated flexion-extension as well as abduction - adduction.

Fingers have at least 6 muscles and the index finger is controlled by 7. Starting with the flexors, the index finger has two flexors the Flexor Digitorum Profundus (FDS) and the Flexor Digitorum Superficialis (FDP). The extensor mechanism consist of the two extensors Extensor Communis (EC) and the Extensor Indicis (EI). There are other 3 muscles involved in the control of the index finger which are the Radial Interosseous (RI), the Ulnar Interosseous (UI) and the Lubricant (LU).

Besides these 7 muscles - tendons that actuate the index finger and produce torques at the 3 joints (active tendons), there are other 4 tendons that we will call passive since they are attached between the other tendons and the bones. These passive tendons are the Terminal Extensor (TE), the Radial Band (RB) the Ulnar Band (UB) and the Extensor Slip (ES). The 4 passive tendons is a way to approximate the extensor mechanism of the fingers. In total for the index finger there are 11 tendons, 7 active and 4 passive.

The basic equation for modeling the tendon excursions is given by the equation that follows:

\[ L = \theta d + 2y \left( 1 - \frac{\theta}{2 tan(\theta/2)} \right) \]  

where \(d\) is the distance from the straight part of the tendon towards the long axis. The term \(y\) corresponds to the distance from the end of the straight part towards the joint centre. This distance is measure along the axis of the bone. A second order polynomial approximation of the equation above is formulated as follows:

\[ L = (b + h\theta)\theta \]  

where \(b\) and \(h\) are constants. The equation above could be used for modeling the excursions of tendons that are involved in flexion - extension as well as for abduction - adduction. A subscript, \(a\), will be used to denote the dependence of the tendon excursion of the abduction - adduction motion, with \(\phi\) being the abduction angle.

In this work we are making use of the model in [1] where (1) is used for the tendon excursions of the flexors FDS and FDP while for the rest of the tendons we are making use of the second order approximation (2). The flexors muscles FDS and FDP depend on the rotations around all the the joints. Since excursions for the flexors is the sum of their excursions of each joint, if (2) were used, there would have been a significance increase of the excursion approximation errors. Therefore 1 is used to model the excursions of the FDS and FDP while the second order approximation 2 is used for the next tendons of the index finger. More precisely we will have

\[ L^{FDP} = \theta d_1^{FDP} + 2y_1^{FDP} \left( 1 - \frac{\theta_1}{2 tan(\theta_1/2)} \right) + \theta d_2^{FDP} + 2y_2^{FDP} \left( 1 - \frac{\theta_2}{2 tan(\theta_2/2)} \right) + \theta d_3^{FDP} + 2y_3^{FDP} \left( 1 - \frac{\theta_3}{2 tan(\theta_3/2)} \right) + \left( h_a^{FDP} + h_a^{FDP} \phi \right) \phi \]  

\[ L^{FDS} = \theta d_1^{FDS} + 2y_1^{FDS} \left( 1 - \frac{\theta_1}{2 tan(\theta_1/2)} \right) + \theta d_2^{FDS} + 2y_2^{FDS} \left( 1 - \frac{\theta_2}{2 tan(\theta_2/2)} \right) + \left( h_a^{FDS} + h_a^{FDS} \phi \right) \phi \]  

The tendon excursion mechanism for EC and TE is rather simple due to their dependence on the rotation of only one joint. The tendon extensor for EC is a function of the rotation at the DIP joint while the tendon excursion of TE is function of the rotation at the PIP joint.

\[ L^{TE} = -r^{TE} \theta_3, \quad L^{ES} = -r^{ES} \theta_2 \]  

The tendon excursions of the RB and UB are functions of the rotation around the PIP joint with the addition of the terminal extensor.

\[ L^{RB} = -\left( b^{RB} + h^{RB} \theta_2 \right) \theta_2 + h^{RB} E^{TE} \]  

\[ L^{UB} = -\left( b^{UB} + h^{UB} \theta_2 \right) \theta_2 + h^{UB} E^{TE} \]  

For the RI the tendon excursion is a function of the MCP rotation only that includes flexion - extension and abduction - adduction. Therefore the tendon excursion is formulated as follows:

\[ L^{RI} = \left( h^{RI} + h^{RI} \theta_1 \right) \theta_1 - \left( h_a^{RI} + h_a^{RI} \phi \right) \phi \]  

Similarly, the tendon excursion for the LI is a function of the MCP rotation but with the addition of the tendon excursion of the UB. Consequently the LI tendon excursion is formulated by the following equation:

\[ L^{LI} = \left( h^{LI} + h^{LI} \theta_1 \right) \theta_1 - \left( h_a^{LI} + h_a^{LI} \phi \right) \phi + L^{UB} \]  

The excursion of the LU tendon is a function of the MCP rotation with the addition of the UB and the subtraction of the
FDP tendon excursions. The excursion of FDP is subtracted from the total excursions of LU since the origin of LU is on FDP. Thus we will have that:

\[
L_{LU} = (b_{LU}^{h} + h_{LU}^{h} \theta_{1}) \theta_{1} - (b_{LU}^{a} + h_{LU}^{a} \phi) \phi + L_{RB} - L_{FDP}^{h}
\]

Finally the tendon excursions of the main extensors of the index finger, EC and EI are function of the MCP rotation and the addition of the displacements that are transformed to the next joints PIP and DIP through the extensor mechanism.

\[
L_{EC} = -rEC \theta_{1} - (b_{EC}^{a} + h_{EC}^{a} \theta_{1}) \theta_{1} + \min(L_{1}, L_{2}, L_{3})
\]

and

\[
L_{EI} = -rEI \theta_{1} - (b_{EI}^{a} + h_{EI}^{a} \phi) \phi + \min(L_{1}, L_{2}, L_{3})
\]

where the terms \(L_{1}, L_{2}\) and \(L_{3}\) are defined as follows:

\[
L_{1} = L^{ES}
\]

\[
L_{2} = L_{UB} + (1 - \beta_{UB}) L^{TE}
\]

\[
L_{3} = L_{RB} + (1 - \beta_{RB}) L^{TE}
\]

In this work we have slightly modified the extensor mechanics for the EI and the EC tendons to avoid the nonlinear operator min by assuming that:

\[
L_{EC} = -rEC \theta_{1} - (b_{EC}^{a} + h_{EC}^{a} \theta_{1}) \theta_{1} + E(L_{1}, L_{2}, L_{3})
\]

\[
L_{EI} = -rEI \theta_{1} - (b_{EI}^{a} + h_{EI}^{a} \phi) \phi + E(L_{1}, L_{2}, L_{3})
\]

with the excursion function \(E(L_{1}, L_{2}, L_{3})\) defined as

\[
E(L_{1}, L_{2}, L_{3}) = \sum_{j=1}^{3} w_{j} L_{j} \text{with } \sum_{j=1}^{3} w_{j} = 1 \text{ and } w_{j} > 0 \forall j = 1, 2, 3
\]

There are 39 parameters for the equations 11 tendons excursions of the index which are provided in table I.

III. INDEX FINGER MOMENT ARM

Since the tendon excursions have been defined for the 11 tendons of the index finger, the moment arm matrix for the 7 active tendons can be found according to the equation:

\[
M(\Theta) = V_{\Theta} L
\]

where \(\Theta = (\theta_{1}, \theta_{2}, \theta_{3}, \phi)^{T}\) and \(L \in \mathbb{R}^{7 \times 1}\) defined as \(L = (L^{FDS}, L^{FDP}, L^{LU}, L^{RI}, L^{EC}, L^{EI})^{T}\). More precisely the moment arm vector for the FDP tendons is expressed as

\[
M_{FDP}^{\theta_{i}} = \begin{bmatrix} M_{FDS}^{\theta_{i}} & M_{FDS}^{\theta_{i}} \end{bmatrix} \text{ where } \forall i = 1, 2, 3
\]

and \(M_{FDP}^{\phi} = h_{a} \phi\). In cases where \(\theta_{i} = 0\) then the moment arm of the FDS is given by the following equation: \(\lim_{\theta_{i} \to 0} M_{FDS}^{\theta_{i}} = d_{FDS}\). The moment arm vector for the FDS tendon is expressed as

\[
M_{FDS}^{\theta_{i}} = d_{i}^{FDS} + y_{i}^{FDS} \begin{bmatrix} \sin(\theta_{i}) - \theta_{i} \\ \frac{2}{2 \sin^{2}(\theta_{i})} \end{bmatrix}
\]

and \(M_{FDS}^{\phi} = 0\). Similarly when \(\theta_{i} = 0\) then the moment arm of the FDS is given by the following equation: \(\lim_{\theta_{i} \to 0} M_{FDS}^{\theta_{i}} = d_{FDS}\). For the LU tendon the moment arm vector is expressed as:

\[
M_{LU}^{\theta_{i}} = \begin{bmatrix} M_{LU}^{\theta_{i}} & M_{LU}^{\theta_{i}} \end{bmatrix} - \begin{bmatrix} h_{a}^{LU} + h_{LU}^{LU} \theta_{1} - M_{FDP}^{h_{a}} \\ M_{RB}^{\theta_{i}} - M_{FDP}^{\theta_{i}} \\ M_{LU}^{\theta_{i}} & M_{FDP}^{\theta_{i}} \end{bmatrix}
\]

Similarly for the UI and RI tendons we will have

\[
M_{UI}^{\theta_{i}} = \begin{bmatrix} M_{UI}^{\theta_{i}} & M_{UI}^{\theta_{i}} \end{bmatrix} = \begin{bmatrix} h_{UI}^{LU} + h_{UI}^{LU} \theta_{1} \\ 0 \end{bmatrix}
\]

\[
M_{UI}^{\phi} = \begin{bmatrix} M_{UI}^{\phi} & M_{UI}^{\phi} \end{bmatrix} = \begin{bmatrix} b_{UI}^{LU} + b_{UI}^{LU} \phi \\ 0 \end{bmatrix}
\]

As we can see from above the moment arm vectors for UI and RI are function of the moment arm vectors of the UB and RB tendons which are defined as follows:

\[
M_{UB}^{\theta_{i}} = \begin{bmatrix} M_{UB}^{\theta_{i}} & M_{UB}^{\theta_{i}} \end{bmatrix} = \begin{bmatrix} 0 \\ -r^{TE} \end{bmatrix}
\]

\[
M_{UB}^{\phi} = \begin{bmatrix} M_{UB}^{\phi} & M_{UB}^{\phi} \end{bmatrix} = \begin{bmatrix} -b_{a}^{LU} + h_{a}^{LU} \phi \\ 0 \end{bmatrix}
\]

\[
M_{RB}^{\theta_{i}} = \begin{bmatrix} M_{RB}^{\theta_{i}} & M_{RB}^{\theta_{i}} \end{bmatrix} = \begin{bmatrix} 0 \\ -r^{TE} \end{bmatrix}
\]

\[
M_{RB}^{\phi} = \begin{bmatrix} M_{RB}^{\phi} & M_{RB}^{\phi} \end{bmatrix} = \begin{bmatrix} -b_{a}^{LU} + h_{a}^{LU} \phi \\ 0 \end{bmatrix}
\]

Fig. 1. Anatomical structure of the index finger [9]
Finally the moment arm vectors of the main extensor tendons EC and EI of the index finger are expressed as follows:

\[
M^E = \begin{bmatrix}
M^E_{\theta_1} & M^E_{\theta_2} & M^E_{\theta_3} & M^E_{\phi}
\end{bmatrix}^T = \begin{bmatrix}
-w_1F_{EC} - w_2(b_{UB} + h_{UBT_2}) - w_3(b_{RB} + h_{RBT_2}) \\
-w_2F_{TE} - w_3F_{TE} \\
-b_{EC} + h_{EC}F
\end{bmatrix}
\]

and

\[
M^E_{\phi} = \begin{bmatrix}
M^E_{\phi_1} & M^E_{\phi_2} & M^E_{\phi_3} & M^E_{\phi}
\end{bmatrix}^T = \begin{bmatrix}
-w_1F_{ES} - w_2(b_{UB} + h_{UBT_2}) - w_3(b_{RB} + h_{RBT_2}) \\
-w_2F_{TE} - w_3F_{TE} \\
-b_{EI} + h_{EI}F
\end{bmatrix}
\]

The moment arm matrix for the active tendons \(M_\Theta\) is therefore defined:

\[
\begin{bmatrix}
M^E_{\phi} & M^E_{\phi} & M^E_{\phi} & M^E_{\phi} & M^E_{\phi} & M^E_{\phi} & M^E_{\phi} & M^E_{\phi} & M^E_{\phi} & M^E_{\phi}
\end{bmatrix}
\]

while the velocity with which the length of the tendons changes over time is given by the equation that follows:

\[
V(\Theta, \dot{\Theta}) = M_\Theta \times \dot{\Theta}
\]

IV. MUSCLE MODEL

In our simplified hill type muscle model [2] the torque is generated by the active tendons FDS, FDP, LU, UI, RI, EC and EI which are the tendons connected to muscles. Therefore the torques are formulated as follows:

\[
\tau = M(\Theta) \cdot T(\alpha, L(\Theta)\cdot V(\theta, \dot{\theta}))
\]

The tension depends on the activation of the muscles but also varies with the velocity \(V = V(\theta, \dot{\theta})\) and the length \(L = L(\theta)\) of that muscle which are function of the configuration and joint velocity of the index finger. Therefore the tension is mathematically formulated as follows:

\[
T(\alpha, L(\Theta), V(\theta, \dot{\theta})) = F_L(L(\theta)) + F_F(V(\theta, \dot{\theta})) + F_P(L(\theta))
\]

where the terms \(F_L(L(\theta))\) and \(F_F(V(\theta, \dot{\theta}))\) are force functions that describe the force - length and force velocity properties of muscles [3]. These force function are defined as follows:

\[
F_L(L(\theta)) = \begin{cases}
0 & \text{if } L < 1 \\
(L(\theta) - 1)^2 L & \text{if } L > 1
\end{cases}
\]

\[
F_F(V(\theta, \dot{\theta})) = \begin{cases}
0 & \text{if } V < 0 \\
\frac{2}{1 + \exp(V(\theta, \dot{\theta})L_0)} & \text{if } V > 0
\end{cases}
\]

The length of the muscles is expressed in units of \(L_0\) where \(L_0\) is the length at which the maximum isometric force is generated [2], [3]. In addition velocity \(V(\theta, \dot{\theta})\) is expressed in units of \(L_0/sec\). The muscle length and velocity are converted into normalized units of \(L_0\) according to the operating length range of each one of the muscles. An illustration of the force-length force-velocity profiles is given in Fig 2.

After specifying the moment arm of the index finger and discussing the muscle model used in this work, in the next section we provide our result on the application of stochastic optimal control for movement generation of the tendon-driven index finger.
the 3D representation of force as function of muscle velocity and length. The stochastic differential equation that follows:

\[ h \] rather general class of dynamical systems which are found in the open and close loop gains stability around the point of linearization of the nonlinear dynamics. Since the open and close loop gains have been specified the next step is the backward propagation of the terms \( s_k, s_{k+1} \) and \( s_{k+1} \). This backward propagation is expressed by the equations that follow:

\[ v^k(x_1) = s_k + s^T_{k+1} \delta x + \delta x^T S_{k+1} \delta x \]
\[ S_k = Q_k + A_k^T S_{k+1} A_k + L_k^T H L_k + L_k^T G + G^T L_k \]
\[ s_k = q_k + A_k^T s_{k+1} + L_k^T H L_k + G^T L_k + L_k^T g \]
\[ s_k = q_k + s_{k+1} + \frac{1}{2} \sigma^2 \sum_i c_{j,k}^T s_{k+1} c_{i,k} + \frac{1}{2} L_k^T H L_k + L_k^T g \]

The control policy at the next iteration is given by the adding the correction \( \delta u_{i,T} \) in the control policy of the current iteration. Therefore we will have that \( u_{i+1,T} = u_{i,T} + \delta u_{i,T} \). Using the updated control policy \( u_{i+1,T} \) and by propagating the nonlinear dynamics a new trajectory is generated in state space. The linear and quadratic approximation of the dynamics and cost are found and the algorithms is repeated again until convergence.

The control law \( \delta u_{i,T} = -H^{-1}(g + G \delta x_k) \) is the optimal one for as long as the matrix \( H \) is positive definite. As we have show in [5],[4] the cost-to-go function \( v_2(\delta x) \) depends on the control law \( \delta u_k = \pi_k(\delta x) \) through the term \( \alpha(\delta x, \delta u) = \delta u^T (g + G \delta x) + \frac{1}{2} \delta u^T H \delta u \). Therefore minimization of the cost to go function is equivalent to the minimization of the quadratic function \( \alpha(\delta x, \delta u) \) which is convex iff the hessian \( H > 0 \). In highly dimensional dynamical systems \( H \) might loose its positive definiteness.

In such cases we follow an approach similar to Levenberg-Marquardt (1): compute the eigenvalue decomposition of \( H \), \( [V,D] = \text{eig}(H)(2) \) replace all the negative elements of the diagonal matrix with \( \delta \) (3) add a small positive number \( \lambda \) to the diagonal of \( D \) (4) set \( H = V D V^T \) using the modified diagonal matrix \( D \) from the steps (2) and (3).

![Fig. 3. The stochastic optimal feedback controller reduces the variability and perturbations MCP rotation.](image)

Finally, to see the efficiency of the stochastic optimal feedback controller we illustrate in figure 3 the effect of noise in the kinematic trajectories of the MCP joint for the open loop and close loop case. In the open loop case only the open loop gain \( L_k \) is applied to the system. In the close loop case both open \( L_k \) and close loop gain \( L_k \) are applied. As we can see, under the presence of perturbation and noise in the dynamics the feedback gain reduces the variability of the trajectories towards the target. Similar behavior is observed for the kinematic trajectories of PIP and DIP joints.

### VI. Multibody Dynamics

The full model of the index finger is given by the equations that follow:

\[ \dot{\theta} = -I(\theta)^{-1} \cdot C(\theta, \theta) + I(\theta)^{-1} \cdot \tau \]
\[ \tau = M(\theta) \cdot T(\alpha_l(\theta), V(\theta, \dot{\theta})) \]
\[ \ddot{a}_i = D \cdot a_i + C \cdot u_i \quad i = 1, ..., 7 \]

where \( I \in \mathbb{R}^{6 \times 6} \) is the inertial matrix and \( C(\theta, \dot{\theta}) \in \mathbb{R}^{6 \times 1} \) is matrix of coriolis and centrifugal forces. For our simulations we have excluded the abduction - addition movement at MCP joint and we examine planar movements and we investigate the necessary length and velocity profiles of the tendons for producing such movements. Therefore, in state space formulation our model has dimensionality equal to 13 states that corresponds to 6 states of the joint space kinematics(angles and velocities) and 7 states for the activation variables of the muscles. The quantities \( \theta \) and \( \dot{\theta} \) are vectors of dimensionality \( \theta \in \mathbb{R}^{3 \times 1}, \dot{\theta} \in \mathbb{R}^{3 \times 1} \) defined as \( \theta = (\theta_1, \theta_2, \theta_3) \) and \( \dot{\theta} = (\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3) \). The inertia \( I(\theta) \) terms of the forward dynamics are given as follows:

\[ I_{11} = I_{31} + \mu_1 + \mu_2 + 2 \mu_4 \cos \theta_2 \]
\[ I_{21} = I_{22} + \mu_4 \cos \theta_2 + \mu_6 \cos (\theta_2 + \theta_2) \]
\[ I_{22} = I_{33} + \mu_2 + 2 \mu_5 \cos \theta_3 \]
\[ I_{31} = I_{32} + \mu_4 \cos (\theta_1 + \theta_3) \]
\[ I_{33} = \mu_3 \]

while the term of coriolis and centrifugal forces \( C(\theta, \dot{\theta}) \) is formulated as follows:

\[ C_1 = \mu_4 \sin \theta_2 \left[ -\theta_2 (2\theta_1 + \theta_3) \right] + \mu_5 \sin \theta_3 \left[ -\theta_1 (2\theta_1 + 2\theta_2 + \theta_3) \right] - \mu_6 \sin (\theta_2 + \theta_2) (\theta_2 + \theta_1) \times (2\theta_1 + \theta_2 + \theta_3) \]
\[ C_2 = \mu_5 \sin \theta_2 \theta_1^2 + \mu_5 \sin \theta_3 \theta_1^2 + \mu_6 \sin (\theta_2 + \theta_1) \theta_1^2 \]
\[ C_3 = \mu_5 \sin \theta_1 (\theta_1 + \theta_2) + \mu_6 \sin (\theta_2 + \theta_3) \theta_1^2 \]
The terms $\mu_1, \mu_2, \mu_3$ are functions of the masses $(m_1, m_2, m_3) = (0.05, 0.04, 0.03) Kgr$ and the lengths $(l_1, l_2, l_3) = (0.0508, 0.0254, 0.01905) m$ of the 3 bones of the index finger.

$$
\begin{align*}
\mu_1 &= (m_1 + m_2 + m_3) l_1^2, \\
\mu_2 &= (m_2 + m_3) l_2^2, \\
\mu_3 &= m_3 l_3^2, \\
\mu_4 &= (m_2 + m_3) l_1 l_2, \\
\mu_5 &= m_3 l_1 l_3.
\end{align*}
$$

VII. RESULTS

We apply the iLQG to the model of the index finger for the tapping task. Since for the tapping task we do not assume any desired trajectory the immediate cost function $\ell (\tau, x(\tau), \pi (\tau, x(\tau)))$ is only function of the controls and not function of the state. Therefore it is formulated as:

$$
\ell (\tau, x(\tau), \pi (\tau, x(\tau))) = \pi^T R \pi + \text{control cost weight matrices } R_0 = r I_{2 \times 2} \text{ and } r = 0.0001. \text{ The terminal cost } h(x(T)) \text{ is specified as } h(x(T)) = (\theta - \theta^*)^T Q (\theta - \theta^*) \text{ with } Q = I_{3 \times 3}. \text{ The parameter } \theta^* = (-60^\circ, 45^\circ, 15^\circ) \text{ is the target configuration of the index finger just before contact occurs while the initial configuration is } \theta_0 = (60^\circ, -90^\circ, -18^\circ).
$$

Fig. 4. Tendon excursion profile for the 0.3 sec tapping movement starting from the initial configuration $\theta_0 = (60^\circ, -90^\circ, -18^\circ)$ to target configuration $\theta^* = (-60^\circ, 45^\circ, 15^\circ)$. For tapping movement the FDS,RI,UI and LU tendons flex and therefore their length decreases during tapping. The main extensors of the index finger EC and EI increase it. Moreover, the FDP tendon has velocity that is negative and therefore its length decreases. The fact that length decrease is very small is due to 1) its size - length that is in general much bigger than the rest of the tendons and therefore small changes are not significant w.r.t its rest length 2) the tapping task starting from the initial configuration $\theta = (60^\circ, \pi/2, -\pi/10)$ to target configuration $\theta^* = (-\pi/6, -\pi/4, -\pi/12)$ does not require the involvement of the FDP tendon. Obviously this observation is not valid for all possible movements. Finally figure 7 illustrates the changes in length of the passive tendons TE,ES, RB and UB as well as the torques applied to the MCP, PIP and DIP joints of the index finger while figures 6 shows the sequence of postures during tapping.

In figure (4) the excursions of the active tendons are illustrated for the time horizon of 0.3 sec during tapping. As we can see the FDS,RI,UI and LU tendons flex and therefore their length decreases during tapping. The main extensors of the index EC and ES increase their length as it is expected for the tapping task. Moreover even though for the FDP the length appears to be constant, it is actually reduced during tapping. This reduction is small with respect the decrease of the excursions of the rest tendons and therefore it can not be properly illustrated.

In figure (5) the velocity profiles of the active tendons are illustrated. More precisely we can see that the velocities of the tendons FDS,RI,UI and LU are negative while the velocities of the extensors EC and EI are positive and they increases during the time horizon of 0.3 sec. This observation is in agreement with our previous result in figure (4) where we find that FDS,RI,UI and LU reduce their length while EC and EI increase it. Moreover, the FDP tendon has velocity that is negative and therefore its length decreases. The fact that length decrease is very small is due to 1) its size - length that is in general much bigger than the rest of the tendons and therefore small changes are not significant w.r.t its rest length 2) the tapping task starting from the initial configuration $\theta = (60^\circ, \pi/2, -\pi/10)$ to target configuration $\theta^* = (-\pi/6, -\pi/4, -\pi/12)$ does not require the involvement of the FDP tendon. Obviously this observation is not valid for all possible movements. Finally figure 7 illustrates the changes in length of the passive tendons TE,ES, RB and UB as well as the torques applied to the MCP, PIP and DIP joints of the index finger while figures 6 shows the sequence of postures during tapping.

Fig. 5. Velocity profiles of the active tendons during tapping. The profiles are in agreement with the corresponding tendons excursions. More precisely the tendons velocity of FDS,RI,UI,LU is negative and therefore that leads to decrease of the corresponding excursions. Similarly, the velocity for the two main extensor of the index finger EC and EI is positive and thus the corresponding excursions increase as well.

Fig. 6. The sequence of postures during tapping for the index finger.

For the task of tapping with zero terminal velocity, figures 8 illustrates the length and velocity of the active tendons while figure 9 illustrate the length of the passive tendons and the underlying torques applied to the 3 joints. As expected, the velocity of the active tendons decreases toward zero at the end of the time horizon while the corresponding length remain almost constant for same time interval.
In this work we have applied the stochastic optimal feedback control framework to a full tendon driven model of the index finger. We have considered all the tendons of the index finger while we have taken into account the force length and force velocity properties of muscles. Our stochastic optimal controller provides the complete time history of the tendon excursions and tendon velocities of the finger for the two tasks of tapping with nonzero and zero velocity at the point of contact. Moreover our simulations suggest that stochastic feedback control can successfully handle the highly nonlinear dynamics of the index finger which are resulting from the addition of tendon and muscle model. To our knowledge this paper is the first presentation of the use of optimal control to actuate individual muscles with physiological properties to produce a trajectory that is not prespecified.

B. Future Work

After having applied the iLQG controller to the biomechanical model of the index finger there are many future direction that we would like to consider for our research. More precisely on the robotics side, we are currently working for the extension of the current framework to the 3D movement of the index finger based on the specification and the geometry of the finger. For this extensions the abduction - adduction rotation at MCP joint has to be considered while the dimensionality of the dynamics will increase to 15 states. The ultimate goal of this extension is to apply the stochastic optimal feedback control to an anatomically corrected testbed robotic finger [10] for a variety of movement tasks.

On the biomechanical side we can use the generated tendon length and velocity profiles for comparison with experimental data. Unfortunately, technology on recording and extracting the tendon excursions from human subjects with no invasive techniques is still under development and it faces many challenges. However we believe that in the near future we will be in the position to make such comparisons. Currently we are also working towards applying the stochastic optimal feedback control in experiments with cadaver fingers.

VIII. CONCLUSIONS AND FUTURE WORK

A. Conclusions

In this work we have applied the stochastic optimal feedback control framework to a full tendon driven model of the index finger. We have considered all the tendons of the index finger while we have taken into account the force length and force velocity properties of muscles. Our stochastic optimal controller provides the complete time history of the tendon excursions and tendon velocities of the finger for the two tasks of tapping with nonzero and zero velocity at the point of contact. Moreover our simulations suggest that stochastic feedback control can successfully handle the highly nonlinear dynamics of the index finger which are resulting from the addition of tendon and muscle model.

Fig. 7. The time history of the length of the passive tendons and the torques for the tapping task of the index finger with nonzero terminal velocity.

Fig. 8. The time history of the length and velocity of the active tendons of the index finger for the tapping task with zero terminal velocity.

Fig. 9. The time history of the length of the passive tendons and the torques of the index finger for the tapping task with zero terminal velocity.

REFERENCES