CS545—Contents IV

- Frequency Domain Representations
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- Matlab/Simulink Introduction
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- Reading Assignment for Next Class
  - See http://www-clmc.usc.edu/~cs545
The Laplace Transform

- Properties of Frequency Domain Representations
  - A convenient method so solve (linear!) differential equations (even without a computer …) by converting them to algebraic equations
  - Makes system analysis easy, even for very big systems
  - Simple mathematics
  - Only applicable for linear time invariant systems!
  - Analyzes signals in terms of sinusoids and exponentials (includes Fourier transforms as special case)

Pierre-Simon, Marquis de Laplace 1749-1827
French Mathematician
The Laplace Transform

- The Core of Frequency Domain Analysis: The Laplace Transform

\[ L(f(t)) = f(s) = \int_0^\infty f(t)e^{-st} \, dt \]

where

\[ s = \sigma + j\omega \quad \text{and} \quad j = \sqrt{-1} \]
The Laplace transform. The Laplace transform converts a signal in the time domain, \( x(t) \), into a signal in the s-domain, \( X(s) \) or \( X(F,T) \). The values along each vertical line in the s-domain can be found by multiplying the time domain signal by an exponential curve with a decay constant \( F \), and taking the complex Fourier transform. When the time domain is entirely real, the upper half of the s-plane is a mirror image of the lower half.
Waveforms associated with the s-domain. Each location in the s-domain is identified by two parameters: \( \sigma \) and \( \omega \). These parameters also define two waveforms associated with each location. If we only consider pairs of points (such as A & A', B & B', and C & C'), the two waveforms associated with each location are sine and cosine waves of frequency \( \omega \), with an exponentially changing amplitude controlled by \( \sigma \).
Most Important Laplace Transforms

\[ L(ax(t)) = aL(x(t)) \quad \text{where } a \text{ is a constant} \]
\[ L(x(t)) = x(s) \]
\[ L(u(t)) = u(s) \]
\[ L(\dot{x}(t)) = sx(s) - x(0) \quad \text{(commonly, } x(0) = 0 \text{, accomplished by coordinate transformations)} \]
\[ L(\ddot{x}(t)) = s^2x(s) \quad \text{(and analogues for higher derivatives)} \]
\[ L(\int x(t)dt) = \frac{1}{s}x(s) \]
Transfer Functions

- The Transfer Function describes the Input-Output Relationship of a dynamical system:

\[ x(s) = H(s)u(s) \]

- Example I:

  Time Domain:
  \[ \ddot{x} = -b\dot{x} - kx + u \]

  Frequency Domain:
  \[ s^2 x(s) = -bsx(s) - kx(s) + u(s) \]
  \[ x(s) = \frac{1}{s^2 + bs + k} u(s) = H(s)u(s) \]
Transfer Functions (cont’d)

- Example II: An Integrator

\[ \dot{x} = u \]

\[ sx(s) = u(s) \quad \Rightarrow \quad x(s) = \frac{1}{s}u(s) \]

- Example III: A Simple Low Pass Filter

\[ \dot{x} = \alpha(u - x) \]

\[ sx(s) = -\alpha x(s) + \alpha u(s) \quad \Rightarrow \quad x(s) = \frac{\alpha}{s + \alpha}u(s) \]
Transfer Functions (cont’d)

- Example IV: A negative Feedback System

\[
\dot{x} = ax + bu = ax + bk(x_d - x)
\]

\[
sx(s) = ax(s) + bk(x_d(s) - x(s)) = ax(s) + bkx_d(s) - bkx(s)
\]

\[
x(s) = \frac{bk}{s - a + bk} x_d(s)
\]
Block Diagram Algebra

\[ H(s) = H_1(s) + H_2(s) \]

\[ H(s) = H_2(s)H_1(s) \]

\[ H(s) = H_1(s)(I + H_2(s)H_1(s))^{-1} \]
Matlab/Simulink Simulations

- An Example