CS545–Introduction to Robotics

Homework Assignment 3 (Due May 1)

In the following problems, you should use MATLAB to compute numerical results and visualize the data, and Simulink for simulations. A handout about getting started with MATLAB is in

http://www-clmc.usc.edu/~cs545/homework.php

This web page also contains all the files needed below. IMPORTANT: In your solutions of the homework, also provide intermediate steps how you derived the solution to a problem.

1. (100 Points) In the lecture on movement planning, we mentioned dynamic systems approaches as a possible way to generate movement plans out of the time evolution of nonlinear differential equations. The following set of differential equations realize a general way of generating point-to-point movements:

Transformation System:
\[
\dot{z} = \alpha_z (\beta_z (g - y) - z) + f
\]
\[
\dot{y} = z
\]

Canonical System:
\[
\dot{x} = -\alpha_x x
\]

Nonlinear Function:
\[
f = \sum_{i=1}^{N} \frac{\sum_{j=1}^{N} \psi_i(x) w_j}{\sum_{j=1}^{N} \psi_j(x)} \cdot x (g - y_0) = \varphi^T w
\]

where

\[
\varphi = \begin{bmatrix}
\psi_1(x) \\
\sum_{j=1}^{N} \psi_j(x)
\end{bmatrix}
\]
\[
\psi_i(x) = \exp \left( -\frac{1}{2\sigma_i^2} (x - c_i)^2 \right)
\]

The Transformation System is a simple damped spring system in first-order notation, with an additive nonlinear term $f$. The goal position of the system is given by $g$, the position is $y$, the velocity is $\dot{y}$, and the acceleration is $\ddot{y} = \dot{z}$. The symbols $\alpha_z, \beta_z$ are positive time constants.
The Canonical System is a timing signal. Usually, the state $x$ is initialized to 1, and the first order differential equation with positive time constant $\alpha$, creates an exponential decay of $x$ to zero over time.

The Nonlinear Function $f$ is realized by a sum of Gaussian basis functions $\psi_i$, which are located at centers $c_i$ and have width determined by $\sigma_i^2$. $y_0$ is the start position of $y$, i.e., $y(t = 0) = y_0$. The weights $w_i$ determine the shape of the nonlinear function $f$.

Equations (1) are called a dynamic movement primitive (DMP). The triple $y, \dot{y}, \ddot{y}$ is interpreted as a desired trajectory for one degree-of-freedom (DOF) of the robot. In this problem, we will analyze some of the properties of DMPs.

a) What is the equilibrium point $y$ of the DMP? Provide your derivation and result.

b) Is the equilibrium point stable? Why? Derive the stability analysis in symbolic form and base your argument on eigenvalues. (Hint: there are two ways of approaching the stability analysis. First, you could perform a full linear stability analysis of the linearized system as in Lecture 6. But this becomes quite complex. More suitably, think about which value the nonlinear function $f$ has at the equilibrium point. This knowledge allows you to analyze a simpler system in terms of its stability properties.)

c) Assume the following numerical values of the time constants and initial conditions:

\[
\alpha_z = 25 \\
\beta_z = 6 \\
\alpha_x = 8 \\
y_0 = 0 \\
x(t = 0) = 1 \\
z(t = 0) = 0 \\
N = 10 \\
g = 1 \\
c = [1.0000 0.6294 0.3962 0.2494 0.1569 0.0988 0.0622 0.0391 0.0246 0.0155] \\
\sigma^2 = [41.6667 16.3934 6.5359 2.5840 1.0235 0.4054 0.1606 0.0636 0.0252 0.0252]/1000 \\
w = [0 0 0 0 0 0 0 0 0 0] \\
\]

Write a Matlab program that integrates these differential equations with Euler Integration using a time step of $dt=0.001$ from $t=0$ to $t=1$ second (i.e., 1000 integration time steps). Provide a plots of i) $y, \dot{y}, \ddot{y}$ as a function of time, ii) $x$ as a function of time, iii) $\psi_i$ as a function of time (try to plot of 10 $\psi_i$ in one plot).

d) The weight vector $w$ allows you to create a large variety of different time courses to the goal $g$. Manually create three different weight vectors such that you get “interesting” different trajectories for $y, \dot{y}, \ddot{y}$. Provide the same plots as in c) for these different weight vectors.

e) It is also possible to compute weight vector $w$ from a demonstrated trajectory, given to you by a teacher. This is also called imitation learning. For this purpose, assume that you are given a time series of data $y(t), \dot{y}(t), \ddot{y}(t)$ from $t=0$ to $t=1$, with time resolution of $dt=0.001$ (i.e., you get 1001 data points that describe the position, velocity, and acceleration trajectory). As indicated in Equation (1), the non-
linear function $f$ can be written as a linear function in $w$ with basis function vector $\phi$. Rearrange the Transformation System such that it has the standard appearance of a linear regression problem. For linear regression and multiple data points, the general regression problem looks like $t = Xw$. Write down how the rows of $X$ and $t$ are computed for imitation learning with DMPs. Write down the regression solution for $w$ in general symbolic form using $X$ and $t$. Download the file imitation.data from the homework web page: it has three columns and 1001 data points, where the columns contain $y(t), \dot{y}(t), \ddot{y}(t)$ of a demonstrated trajectory, and the rows are ordered in time, starting with $t=0$, and ending with $t=1$. Compute the weight vector $w$ for this data set, and generate the same plots as in c) for this weight vector. Comment on the quality of the imitation by comparing the training data with the data generated by the DMP.