Inertial Sensor-Based Humanoid Joint State Estimation
Nicholas Rotella1  Sean Mason1  Stefan Schaal12  Ludovic Righetti2
1CLMC Lab, University of Southern California
2Autonomous Motion Department, Max-Planck Institute for Intelligent Systems

Motivation
Humanoids use one IMU for base pose estimation
High-quality IMUs have become cheap and common
What can we estimate with many IMUs?

Joint State Estimators
Define state \( x = [\dot{\theta}^T, \ddot{\theta}^T]^T \) so

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\dot{\theta} \\
\ddot{\theta}
\end{bmatrix} +
\begin{bmatrix}
0 \\
1
\end{bmatrix}
\begin{bmatrix}
\bar{R}_W \\
\bar{a}
\end{bmatrix}
\]

Joint acceleration \( \bar{\theta} \) given by
\[
\bar{\theta} = \left( J(\theta, \dot{\theta}, \ddot{\theta}) \right) \bar{R} - n(\theta, \dot{\theta})
\]

Measurement: joint angles and IMU-based velocities
Also estimate time-varying gyroscope biases

IMU-Based Feedback Control
PD control task switched between numerical and IMU-based velocities
Reduced RMS error from 0.0103rad to 0.0099rad in position and 0.3786rad/s to 0.0902rad/s in velocity.
Able to increase feedback gains by 50%
Future work: use IMU-based joint velocity estimates in a whole-body controller

Goals
Joint derivatives computed numerically from noisy sensors, filtering induces delays
Compute derivatives directly from inertial sensors
Account for unknown link IMU poses
Increase control authority/damping using IMU signals

Attached one Microstrain 3DM-GX3-25 IMU per link.
3-Axis Gyroscope: \( \bar{\omega}_{\text{IMU}} = R_W^\theta \omega_W^\theta \)
3-Axis Accelerometer: \( \bar{a}_{\text{IMU}} = R_W^\theta (a_W^\theta + g) \)

Joint Velocities from Gyroscopes
Computed using either relative link poses or link Jacobians which provide constraints.
Develop automatic IMU orientation calibration to account for unknown offsets.

Joint Accelerations from Accelerometers
Requires knowledge of local IMU position automatically determined from calibration.
Gravity cancels out (global information not required)
Prove that three IMUs adjacent to joint necessary for solution

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>( \dot{\theta}_{-1} )</th>
<th>( \ddot{\theta}_{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>2</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td>4</td>
<td>0.05</td>
<td>0.06</td>
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<tr>
<td>6</td>
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<td>0.08</td>
</tr>
<tr>
<td>8</td>
<td>0.09</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Joint IMU
Filtered
Filtered, with acc

 desserts

IMU
W
IMU
W
\( \bar{R} \)
\( \bar{a} \)

Setup
Computational Learning and Motor Control Lab, USC
Numerical Velocity IMU-Based Velocity
http://www-clmc.usc.edu/Main/NickRotella
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Joint Velocities from Gyroscopes

Can express angular velocity measured by any link IMU in terms of measured angular velocities of preceding links and joint velocities.

- **Link 1 IMU (Base):**
  \[ \dot{\theta}_1 = R_0^W \dot{\omega}_W = \theta_0 \]

- **Link 2 IMU:**
  \[ \dot{\theta}_2 = R_0^W \dot{\omega}_W = R_0^W (\dot{\omega}_W + \bar{\omega}_W) \]
  \[ = R_0^W \dot{\omega}_W + R_0^W \bar{\omega}_W \]

Using constrained kinematics:

\[ \dot{\theta}_1 = \theta_0 \]
\[ \dot{\theta}_2 = \theta_1 \]

In general:

\[ \dot{\theta}_i = \sum_{i=1}^{n} R_i^W \dot{\omega}_W \]

**Motivation:**

- Humanoids use one IMU for base pose estimation
- High-quality IMUs have become cheap and common
- What can we estimate with many IMUs?

Joint Accelerations from Accelerometers

Again, can express measured linear accelerations of any link IMU in terms of the measured accelerations of preceding link IMUs and joint accelerations.

- **Link i - 1 IMU:**
  \[ \ddot{a}_{i-1} = R_i^W (a_{i-1} + g) \]
  \[ \dot{\theta}_{i-1} = \theta_{i-1} \]

- **Link i IMU:**
  \[ \ddot{a}_i = R_i^W (a_i + g) = R_i^W (a_{i-1} + \dot{\theta}_{i-1} + \ddot{a}_{i-1}) \]
  \[ a_{i-1} = (\dot{\theta}_{i-1} \times a_{i-1}) + \ddot{a}_{i-1} \]
  \[ \dot{\theta}_i = R_i^W \dot{\omega}_W \]

Using the definition of \( a_i \):

\[ \ddot{a}_i = R_i^W \ddot{a}_{i-1} + \dot{\theta}_i \times R_i^W \dot{a}_{i-1} \]

**Note that gravity cancels, assuming accurate kinematics.** Eventually:

\[ \ddot{a}_i = R_i^W (a_{i-1} + (\dot{\theta}_i \times a_{i-1})) \]

Rearranging the previous result,

\[ (r_{i-1})^T \bar{\theta}_{i-1} = R_i^W \ddot{a}_{i-1} - \dot{\theta}_i \times R_i^W \dot{a}_{i-1} + (\dot{\theta}_i \times a_{i-1}) \]

- However, cannot solve because \((r_{i-1})^T\bar{\theta}_{i-1}\) rank-deficient!

**Goals:**

- Joint derivatives computed numerically from noisy potentiometers, filtering induces delays → Compute derivatives directly from inertial sensors
- Account for unknown link IMU poses
- Increase control authority/damping using IMU signals

**Link IMU Pose Calibration**

- Consider effect of noise sources on joint velocity computation:
  \[ T_J(\dot{\theta}) = \dot{\theta} - b - w \]

- Choose state \([\theta^T, \dot{b}^T]^T\), dynamics are
  \[ \dot{\theta} = -T_J(\dot{\theta})^{-1}b + T_J(\dot{\theta})^{-1}(\dot{\theta} - w) \]
  \[ b = w_b \]

- Measure joint positions:
  \[ y = [I \ 0] \begin{bmatrix} \dot{\theta} \\ b \end{bmatrix} + v \]

**Orientation Calibration:**

- All links have velocity \( R_i \dot{\theta}_i = R_i^W \dot{\omega}_W \)
- Transpose, stack \( M \) measurements:
  
  \[ \begin{bmatrix} \hat{\alpha}_1^1 \\ \hat{\alpha}_2^1 \\ \vdots \\ \hat{\alpha}_n^1 \end{bmatrix} \]

- Using \( \hat{a}_i^W = \hat{\alpha}_i^W + g \), obtain:
  
  \[ (\hat{\alpha}_i^W)^2 + \hat{\alpha}_i^W = (\hat{\alpha}_i^W)^2 + \hat{\alpha}_i^W \]

- Stack \( M \) measurements to form:

**Setup**

**Attached one Microstrain 3DM-GX3-25 IMU per link.**

- **3-Axis Gyroscope:**
  \[ \bar{\omega}_{IMU} = R_W^W \dot{\omega}_{W} \]

- **3-Axis Accelerometer:**
  \[ \bar{a}_{IMU} = R_W^W (a_{IMU} + g) \]

**Grospy Bias Estimator**

- Compute orientation and position corrections in base frame (denoted \( R_i(\cdot, r_i) \) assuming leg acts as a rigid body. Achieve this by “locking” leg using stiff joint controllers and physically rotating robot.

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