Inertial Sensor-Based Humanoid Joint State Estimation

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Motivation

- Many humanoids use a single IMU for base pose estimation
- Quality IMUs now relatively cheap/common
- What can we estimate with many IMUs on a robot?
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- Joint angle derivatives computed numerically
- Noise in measured angle necessitates filtering → time delay
- Idea: compute joint velocity and acceleration directly from devices which measure quantities on same order.
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Experimental Setup

Microstrain 3DM-GX3-25 IMU

- **3-Axis Gyroscope:**
  \[ \vec{\omega}_{IMU} = R_{W}^{IMU} \vec{\omega}_{W}^{IMU} \]

- **3-Axis Accelerometer:**
  \[ \vec{a}_{IMU} = R_{W}^{IMU} (\vec{a}_{W}^{IMU} + g) \]
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Joint Velocities from Gyroscopes

Express angular velocity measured by each IMU in terms of velocities of preceding joints in chain using kinematics.

- **Joint velocity:**
  \[
  \dot{\theta}_i = R_{W}^{i-1} \omega_{i-1,i}^W
  \]

- **Link \(i - 1\):**
  \[
  \bar{\omega}_{i-1} = R_{W}^{i-1} \omega_{i-1}^W
  \]

- **Link \(i\):**
  \[
  \bar{\omega}_i = R_{W}^i \omega_i^W
  = R_{W}^i (\omega_{i-1}^W + \omega_{i-1,i}^W)
  = R_{i-1}^i R_{W}^{i-1} \omega_{i-1}^W + R_{i-1}^i R_{W}^{i-1} \omega_{i-1,i}^W
  \]
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Joint Velocities and Accelerations from IMUs

- Solve the system:

\[
\begin{bmatrix}
I & 0 & 0 & \cdots & 0 \\
R_1^2 & I & 0 & \ddots & \vdots \\
R_1^3 & R_2^3 & \ddots & \ddots & \vdots \\
\vdots & \vdots & \ddots & I & 0 \\
R_1^N & R_2^N & \cdots & R_{N-1}^N & I \\
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_0 \\
\dot{\theta}_1 \\
\vdots \\
\dot{\theta}_{N-2} \\
\dot{\theta}_{N-1} \\
\end{bmatrix} =
\begin{bmatrix}
\bar{\omega}_1 \\
\bar{\omega}_2 \\
\vdots \\
\bar{\omega}_{N-1} \\
\bar{\omega}_N \\
\end{bmatrix}
\]

- Same idea for joint acceleration from accelerometers
- Can automatically calibrate IMU pose relative to link
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  R_1^3 & R_2 & \ddots & \ddots & \vdots \\
  \vdots & \vdots & \ddots & I & 0 \\
  R_1^N & R_2^N & \cdots & R_{N-1}^N & I
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In this work, we also:

- Estimate time-varying gyroscope biases
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Using IMU-based joint velocity, can increase feedback gain by 50% before instability, improving tracking.
Come chat with me for details! (Paper TuDbT2.1)