CS545—Contents IV

• Frequency Domain Representations
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  ♦ Most important Laplace Transforms
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  ♦ How to get started
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• Reading Assignment for Next Class
  ♦ See http://www-slab.usc.edu/courses/CS545
The Laplace Transform

• Properties of Frequency Domain Representations
  + A convenient method so solve (linear!) differential equations (even without a computer …)
  + Makes system analysis easy, even for very big systems
  + Simple mathematics
  + Only applicable for linear time invariant systems!

• The Core of Frequency Domain Analysis: The Laplace Transform

\[ L(f(t)) = f(s) = \int_{0}^{\infty} f(t)e^{-st} \, dt \]

where
\[ s = \sigma + j\omega \quad \text{and} \quad j = \sqrt{-1} \]

Magnitude Phase
Most Important Laplace Transforms

\[ L(a) = a \quad \text{where } a \text{ is a constant} \]
\[ L(x(t)) = x(s) \]
\[ L(u(t)) = u(s) \]
\[ L(\dot{x}(t)) = sx(s) - x(0) \quad (\text{commonly, } x(0) = 0, \text{ accomplished by coordinate transformations}) \]
\[ L(\ddot{x}(t)) = s^2 x(s) \quad (\text{and analogues for higher derivatives}) \]
\[ L(\int x(t) \, dt) = \frac{1}{s} x(s) \]
Transfer Functions

• The Transfer Function describes the Input-Output Relationship of a dynamical system:

\[ x(s) = H(s)u(s) \]

• Example 1:

Time Domain:
\[ \ddot{x} = -bx - kx + u \]

Frequency Domain:
\[ s^2 x(s) = -bsx(s) - kx(x) + u(x) \]

\[ x(s) = \frac{1}{s^2 + bs + k} u(s) = H(s)u(s) \]
Transfer Functions (cont’d)

• Example II: An Integrator

\[
\dot{x} = u \\
 sx(s) = u(s) \Rightarrow x(s) = \frac{1}{s} u(s)
\]

• Example III: A Simple Low Pass Filter

\[
\dot{x} = \alpha(u - x) \\
 sx(s) = -\alpha x(s) + \alpha u(s) \Rightarrow x(s) = \frac{\alpha}{s + \alpha} u(s)
\]
Transfer Functions (cont’d)

• Example IV: A negative Feedback System

\[ \dot{x} = ax + bu = ax + bk(x_d - x) \]

\[ sx(s) = ax(s) + bk(x_d(s) - x(s)) = ax(s) + bkx_d(s) - bkx(s) \]

\[ x(s) = \frac{bk}{s - a + bk} x_d(s) \]
Block Diagram Algebra

\[ H(s) = H_1(s) + H_2(s) \]

\[ H(s) = H_2(s)H_1(s) \]

\[ H(s) = H_1(s)(I + H_2(s)H_1(s))^{-1} \]