• Interaction Control
  – Compliance
  – Impedance
  – Force control
  – Hybrid control
  – Impedance control

• Sensors and Actuators

• Reading Assignment for Next Class
  • See http://www-slab.usc.edu/courses/CS545
Problems of Interaction Control

• Equations of motion change: Closed loop kinematic chains
• Motion constraints imposed by the environment: Not all movement plans are feasible anymore
• What are the generalized coordinates?
• Planning and execution usually require very high accuracy if only motion control is performed
  + Exact models of the robot are needed
  + Exact models of the environment are needed
• Thus, somehow it is necessary to control the interaction forces
Some Technical Terms

• Stiffness
  + Proportionality constant $k$ that relates a static displacement to the force due to this replacement
  \[ F = k \Delta x \]

• Compliance
  + Inverse of stiffness
    – Active compliance (or stiffness)
      + Controlled compliance in response to an external force, e.g., in order to keep the contact force at a certain limit (“actively giving in”)
    – Passive compliance (or stiffness)
      + Non-actuated (“internal”) tendency of a body get displaced due to external forces (e.g., bending)

• Impedance
  + Dynamic response to an external force due to inertial, friction, and position terms, i.e.,
  \[ m \ddot{x}_d + b \dot{e} + k e = F \quad \text{where} \quad e = x_d - x \]
Force Control

• In the direction of the constraint, it is more appropriate to do force control than position control

• A simple example: A spring-mass system

\[ \begin{align*}
\text{m} & \quad \text{f} \\
& \quad \text{ke} \\
& \quad x
\end{align*} \]

• What we want to control is the force acting on the environment
Force Control (cont’d)

• Force on the environment:
  \[ f_e = k_e x \]

• Equations of motion:
  \[ f = m \ddot{x} + k_e x + f_{dist} \]

• Reformulate in terms of the variable we want to control
  \[ f = \frac{m}{k_e} \ddot{f}_e + f_e + f_{dist} \]

• Define the error in force
  \[ e_f = f_d - f_e \]

• And generate a control law:
  \[ f = \frac{m}{k_e} (\ddot{f}_d + k_{vf} \dot{e}_f + k_{pf} e_f) + f_e + f_{dist} \]

• Insert control law in eqns of motion results in the error dynamics
  \[ \ddot{e}_f + k_{vf} \dot{e}_f + k_{pf} e_f = 0 \]
• Problems with the suggested control law:
  + Disturbance force is not known
  + Force sensors are quite noisy, such that the derivatives of sensed forces are hard to obtain

• Dealing with the missing disturbance force:
  – Analyze the control law without the disturbance force:
    \[
    \ddot{f} + k_v \dot{f} + k_p f = \frac{k_e}{m} f_{\text{dist}}
    \]
  – Steady state error:
    \[
    e_f = \frac{k_e}{k_p m} f_{\text{dist}}
    \]
    + If \( k_e \) is large, as usually the case in many contact tasks, this error can be quite large
Another control law can improve the steady state error:

\[ f = \frac{m}{k_e} (\ddot{f}_d + k_{vf} \dot{f}_f + k_{pf} e_f) + f_d \]

Insert into equation of motion:

\[
\frac{m}{k_e} (\ddot{f}_d + k_{vf} \dot{f}_f + k_{pf} e_f) + f_d = \frac{m}{k_e} \ddot{f}_e + f_e + f_{dist}
\]

\[
\frac{m}{k_e} (\ddot{f}_f + k_{vf} \dot{f}_f + k_{pf} e_f) + e_f = f_{dist}
\]

\[
\ddot{f}_f + k_{vf} \dot{f}_f + e_f \frac{k_{pf}}{m} = \frac{k_e}{m} f_{dist}
\]

Thus, the steady state error becomes:

\[
e_f = \frac{k_e}{k_{pf} + \frac{k_e}{m}} f_{dist} = \frac{1}{\frac{k_{pf} m}{k_e} + 1} f_{dist}
\]

For stiff environments, this is quite an improvement.
Force Control (cont’d)

• Avoiding force derivatives:
  – As we know:
    \[ f_e = k_e x \]
  – We can make use of:
    \[ \dot{f}_e = k_e \dot{x} \]
    + This assumes that we have good sensors to obtain the velocity of the endeffector

• The final control law thus becomes

\[
 f = \frac{m}{k_e} (\dot{f}_d + k_v (\dot{f}_d - k_e \dot{x}) + k_p f_e) + f_d
\]

• For static desired contact forces we get:

\[
 f = m -k_v \dot{x} + \frac{k_p}{k_e} e_f + f_d
\]

+ Note that we still need to know \( k_e \)
Force Control In A Complete Robot

• For force control, an operational space controller needs to be employed that includes inverse dynamics compensation:
  – The general rigid body dynamics equation is:
    \[ B(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = u \]
  – Try an inverse dynamics control law
    \[ u = B(q)y + C(q, \dot{q})\dot{q} + G(q) + J^T(q)f \]
    + But what is \( y \)? There is no desired acceleration term!

• Solutions to this problem:
  – Set \( y \) to zero
    + This should not matter too much since the interaction motion is usually rather slow or almost static
  – Employ more complex operational control schemes (S&S, Ch.7.4)
Hybrid Control

- The endeffector is usually not constrained in all directions
  - Solution:
    + In task space, use force control in the direction of the constraints
    + Use position control in the unconstrained direction
    + Note: need to transform task controller signals into operational space
  - Example:
    + Wiping clean a window
Impedance Control

- Goal: Control the dynamic response of the endeffector according to a pre-specified second order dynamics system

\[ m\ddot{x}_d + b\dot{e} + ke = F \quad \text{where} \quad e = x_d - x \]

- This means we want to make the robot behave as if it were a different dynamical system

- A possible control law is:

\[ u = B(q)y + C(q,\dot{q})\dot{q} + G(q) - B^{-1}(q)J^T(q)f_e \]

- Where

\[ y = J^{-1}(q)M_d^{-1}(M_d\ddot{x}_d + K_D\dot{\dot{e}} + K_P\dot{e} - M_d\dot{J}(q,\dot{q})\dot{q}) \]

- Which results in the desired error dynamics:

\[ M_d\ddot{e} + K_D\dot{\dot{e}} + K_P\dot{e} = M_dJ^{-T}(q)B(q)J^{-1}(q)f_e \]
Sensors and Actuators

• Elements of an robotic system
  – Power supply
  – Power amplifier
  – Servomotor
  – Transmission
  – Sensors

• Servomotors:
  – Pneumatic
  – Hydraulic
  – Electric motors

• Transmission
  – Gears
  – Pullies and belts or chains
  – No (direct drive)
Sensors

• Sensor types
  – Tactile sensors
  – Proximity sensors
  – Range sensors
  – Vision systems
  – Position sensors (linear and rotary)
  – Velocity sensors (linear and rotary)
  – Acceleration sensors (linear and rotary)
  – Force sensors (linear and torque)

• Analog and Digital sensors exist
CHAPTER 7

INTERACTION CONTROL

One of the fundamental requirements for the success of a manipulation task is the capability to handle interaction between manipulator and environment. The quantity that describes the state of interaction more effectively is the contact force at the manipulator's end effector. High values of contact force are generally undesirable since they may stress both the manipulator and the manipulated object. In this chapter, performance of operational space motion control schemes is studied first. The concepts of mechanical compliance and impedance are defined, with special regard to the problem of integrating contact force measurements into the control strategy. Then, force control schemes are presented which are obtained from motion control schemes suitably modified by the closure of an outer force regulation feedback loop. For the planning of control actions to perform an interaction task, natural constraints set by the task geometry and artificial constraints set by the control strategy are established; the constraints are expressed in a suitable constraint frame. The formulation is conveniently exploited to derive a hybrid force/position control scheme.

7.1 MANIPULATOR INTERACTION WITH ENVIRONMENT

Execution of a manipulation task often requires interaction between manipulator and environment. A complete classification of possible manipulation tasks is practically infeasible in view of the large variety of cases that may occur, nor would such a classification be really useful to find a general strategy to control interaction. Typical examples of manipulation tasks are mechanical part mating, object contour surface tracking, and employment of tools for machining mechanical parts.

During interaction, the environment sets constraints on the geometric paths that can be followed by the manipulator's end effector. This situation is generally referred to as constrained motion.

The use of a purely motion control strategy for controlling interaction requires that the end-effector task trajectory is planned with high accuracy. Also, the control system must guarantee that the end-effector location deviates from the desired location as little as possible along the given trajectory.
Therefore, the success of an interaction task with the environment by using motion control algorithms entirely depends on planning accuracy and control performance. To this purpose, it is crucial to have a detailed model of both manipulator (kinematics and dynamics) and environment (mechanical features and geometry). Manipulator model can be known with enough precision, but a detailed description of the environment is difficult to obtain. Unavoidable occurrence of planning errors may result in a trajectory assigned to the end effector which is no longer suitable for correct task execution.

Another factor that conditions the effectiveness of a motion control approach to the problem of controlling constrained motion is positioning accuracy of the manipulator's end-effector with respect to the environment. There is in fact a lower limit on the accuracy for reproducing the assigned trajectory during task execution, due to both manipulator position errors and uncertainty on the exact position of the environment.

The existence of both modeling errors and finite positioning accuracy affects the determination of the absolute position of the manipulator and environment. A direct consequence is that knowledge of the relative position between the manipulator and environment is affected by uncertainty too. To understand the importance of this implication, it is sufficient to observe that to perform a mechanical part mating with a positional approach, the relative positioning of the parts should be guaranteed with an accuracy of an order of magnitude greater than part mechanical tolerance. Once the absolute position of one part is exactly known, the manipulator should guide the motion of the other with the same accuracy.

The inherent difficulty in exactly planning and controlling manipulator motion necessarily brings up the issue of analyzing the effects of trajectory deviation from the ideal conditions of interaction with the environment.

When the manipulator is governed by position control algorithms, any deviation of the actual trajectory from the reference one provokes a reaction of the control system. This tends to minimize such deviation independently of the generating cause. Hence, if deviation from the planned trajectory is due to interaction of manipulator with environment, reaction forces arise and the position control attempts to reduce the deviation like it would do for any disturbance opposing the end-effector motion. In this case, however, the effect of the control action may be the increase of contact force which is not accompanied by a decrease of deviation. This situation may lead to an increase of contact force until the natural limit set by saturation of manipulator actuators is encountered, or mechanical crisis of one of the elements of interaction takes place.

The higher the environment stiffness and position control accuracy are, the easier an unstable contact case like the one just described can occur. In fact, large constraint reaction forces result from deformation of a stiff environment under a strong position control action.

A measurement of the state of interaction is apparently provided by the features of the contact force exchanged between manipulator and environment. For appropriate handling of interaction it is then necessary to consider control strategies that allow respecting the constraints imposed by the interaction force, either in an indirect way via a suitable use of position control laws or in a direct way via force control laws.
7.2 COMPLIANCE CONTROL

For a detailed analysis of interaction between the manipulator and environment it is worth considering the behavior of the system under a position control scheme when contact forces arise. Since these are naturally described in the operational space, it is convenient to refer to operational space control schemes.

Consider the manipulator dynamic model (6.35). In view of (4.41), the model can be written as

\[ B(q)\ddot{q} + C(q, \dot{q})\dot{q} + F\dot{q} + g(q) = u - J^T(q)h, \]  

(7.1)

where \( h \) is the vector of contact forces exerted by the manipulator's end-effector on the environment.

It is reasonable to predict that, in the case \( h \neq 0 \), the control scheme based on (6.102) no longer ensures that the end effector reaches its desired posture \( x_d \). In fact, by recalling that \( \ddot{x} = x_d - x \), at the equilibrium it is

\[ J_A^T(q)K_P\ddot{x} = J^T(q)h. \]  

(7.2)

On the assumption of a full-rank Jacobian, one has

\[ \ddot{x} = K_P^{-1}T_A^T(x)h = K_P^{-1}h_A, \]  

(7.3)

where \( h_A \) is the vector of equivalent generalized forces that can be defined according to (4.118). Eq. (7.3) shows that at the equilibrium the manipulator, under a position control action, behaves as a generalized spring in the operational space with compliance \( K_P^{-1} \) in respect of force \( h_A \). By recalling the expression of the transformation matrix \( T_A \) in (3.40) and assuming matrix \( K_P \) to be diagonal, it can be recognized that linear compliance (due to force components) is independent of the posture, whereas torsional compliance (due to moment components) does depend on the current manipulator configuration through the matrix \( T_A \).

On the other hand, if \( h \in \mathcal{N}(J^T) \), one has \( \ddot{x} = 0 \) with \( h \neq 0 \), i.e., contact forces are completely balanced by the manipulator mechanical structure; for instance, the anthropomorphic manipulator at a shoulder singularity in Fig. 3.13 does not react to any force orthogonal to the plane of the structure.

For a better understanding of interaction between manipulator and environment, it is necessary to have an analytical description of contact forces. A real contact is a naturally distributed phenomenon in which the local characteristics of both manipulator and environment are involved. In addition, friction effects between parts typically exist which greatly complicate the nature of the contact itself.

A detailed description of the contact is demanding from a modeling viewpoint. To point out the fundamental aspects of interaction control, it is convenient to resort to a simple but significant model of contact. To this purpose, a decoupled elastically compliant environment is considered which is described by the model

\[ h = \begin{bmatrix} \dot{f} \\ \mu \end{bmatrix} = \begin{bmatrix} K_f & 0 \\ 0 & K_{\mu} \end{bmatrix} \begin{bmatrix} \dot{p} \\ \omega dt \end{bmatrix} = K \begin{bmatrix} \dot{p} \\ \omega dt \end{bmatrix}, \]  

(7.4)
where \( dp \) is the vector of translation along the reference frame axes and \( \omega dt \) is the vector of small rotation about the axes of such frame as in (3.83). Hence, the vector \([dp^T \omega^T dt]^T\) describes a generalized displacement from the environment rest position. The stiffness matrix \( K \) is typically positive semi-definite. In fact, the environment does not generate reaction forces along those directions where unconstrained end-effector motion is allowed.

In view of (3.40), Eq. (7.4) can be written in terms of operational space variables as

\[
h = KT_A(x)dx
\]  

(7.5)

where \( dx \) denotes the operational space generalized displacement with respect to the undeformed environment rest position \( x_e \), i.e.,

\[
dx = x - x_e.
\]  

(7.6)

Resorting to (4.118) and (7.6) gives

\[
h_A = T_A^T(x)KT_A(x)dx = K_A(x)(x - x_e)
\]  

(7.7)

that allows relating the equivalent forces on the manipulator with the environment deformation through the matrix \( K_A \), i.e., the environment stiffness matrix. The matrix \( K_A^{-1} \), if it can be defined, is the environment compliance matrix. It represents a passive compliance since it describes an inherent property of the environment in the operational space chosen to express manipulator end-effector position and orientation. By recalling that \( K_A \) is only positive semi-definite, the concept of compliance cannot be globally defined in all operational space but only along those directions, spanning \( \mathcal{R}(K_A) \), along which end-effector motion is constrained by the environment.

On the other hand, notice that the matrix \( K_P^{-1} \) in (7.3) represents an active compliance since it is performed on the manipulator by a suitable position control action. With the environment model (7.7), Eq. (7.3) becomes

\[
\ddot{x} = K_P^{-1}K_A(x)(x - x_e);
\]  

(7.8)

at the equilibrium, the end-effector location is given by

\[
x_\infty = (I + K_P^{-1}K_A(x))^{-1}(x_d + K_P^{-1}K_A(x)x_e),
\]  

(7.9)

while the contact force can be shown to be

\[
h_{A\infty} = (I + K_A(x)K_P^{-1})^{-1}K_A(x)(x_d - x_e).
\]  

(7.10)

Analysis of (7.9) shows that the equilibrium position depends on the environment rest position as well as on the desired position imposed by the control system to the manipulator. The interaction of the two systems (environment and manipulator) is influenced by the mutual weight of the respective compliance features. It is then possible to increase the active compliance so that the manipulator dominates the environment and vice versa. Such a dominance can be specified with reference to the single directions
of the operational space. For a given environment stiffness, according to the prescribed interaction task, one may choose large values of the elements of $K_P$ for those directions along which the environment has to comply and small values of the elements of $K_P$ for those directions along which the manipulator has to comply.

Expression (7.10) gives the value of the contact force at the equilibrium, which reveals that it may be appropriate to tune manipulator compliance with environment compliance along certain directions of the operational space. In fact, along a direction with high environment stiffness, it is better to have a compliant manipulator so that it can taper the intensity of interaction through a suitable choice of the desired position. In this case, the end-effector equilibrium position $x_{\infty}$ practically coincides with the environment undeformed position $x_e$, and the manipulator generates an interaction force, depending on the corresponding element of $K_P$, that is determined by the choice of the component of $(x_d - x_e)$ along the relative direction.

In the dual case of high environment compliance, if the manipulator is made stiff, the end-effector equilibrium position $x_{\infty}$ is very close to the desired position $x_d$, and it is the environment to generate the elastic force along the constrained directions of interest.

In certain cases, it is possible to employ mechanical devices interposed between the manipulator’s end effector and the environment so as to change passive compliance along particular directions of the operational space. For instance, in a peg-in-hole insertion task, the gripper is provided with a device ensuring high stiffness along the insertion direction and high compliance along the other directions (remote center of compliance). Therefore, in the presence of unavoidable position displacements from the planned insertion trajectory, contact forces and moments arise which modify the peg position so as to facilitate insertion.

The inconvenience of such devices is their low versatility to different operating conditions and generic interaction tasks, i.e., whenever a modification of the compliant mechanical hardware is required. On the other hand, with active compliant actions the control software can be easily modified so as to satisfy the requirements of different interaction tasks.
Example 7.1. Consider the two-link planar arm whose tip is in contact with a purely frictionless elastic plane; let \( x_e \) be the equilibrium position of the plane, which is assumed to be orthogonal to axis \( z \) (Fig. 7.1). The environment stiffness matrix is

\[
K_A = K_f = \text{diag}\{k_z, 0\},
\]

corresponding to the absence of interaction forces along the vertical direction \( (f_y = 0) \). Let \( \mathbf{p}_d = [x_d \ y_d]^T \) be the desired tip position, which is located beyond the contact plane. The proportional control action on the arm is characterized by

\[
K_P = \text{diag}\{k_{P_x}, k_{P_y}\}.
\]

The equilibrium equations for position (7.9) and force (7.10) give

\[
\mathbf{p}_\infty = \begin{bmatrix} k_{P_x}x_d + k_z x_e \\ k_{P_x} + k_z \\ y_d \end{bmatrix}, \quad f_\infty = \begin{bmatrix} k_{P_x}k_z(x_d - x_e) \\ k_{P_x} + k_z \\ 0 \end{bmatrix}.
\]

With reference to positioning accuracy, the arm tip reaches the vertical coordinate \( y_d \), since the vertical motion direction is not constrained. As for the horizontal direction, the presence of the elastic plane imposes that the arm can move as far as it reaches the coordinate \( x_\infty \). The value of the horizontal contact force at the equilibrium is related to the difference between \( x_e \) and \( x_d \) by an equivalent stiffness coefficient which is given by the parallel composition of the stiffness coefficients of the two interacting systems. Hence, the arm stiffness and environment stiffness influence the resulting equilibrium configuration. In the case when

\[
k_{P_x}/k_z \gg 1,
\]

it is

\[
x_\infty \approx x_d \quad f_\infty \approx k_z(x_d - x_e)
\]

and thus the arm prevails over the environment, in that the plane complies almost up to \( x_d \) and the elastic force is mainly generated by the environment (passive compliance). In the opposite case

\[
k_{P_x}/k_z \ll 1,
\]

it is

\[
x_\infty \approx x_e \quad f_\infty \approx k_{P_x}(x_d - x_e)
\]

and thus the environment prevails over the arm which complies up to the equilibrium \( x_e \), and the elastic force is mainly generated by the arm (active compliance).

To complete the analysis of manipulator compliance in contact with environment, it is worth considering the effects of a joint space position control law. With reference to (6.43), in the presence of end-effector contact forces, the equilibrium posture is determined by

\[
K_P\ddot{q} = J^T(q)h
\]

and then

\[
\ddot{q} = K_P^{-1}J^T(q)h.
\]
On the assumption of small displacements from the equilibrium, it is reasonable to compute the resulting operational space displacement as \( \ddot{x} = J_A(q)K_p^{-1}J_A^T(q)h_A \) which, in view of (4.118) and (7.12), can be written as

\[
\ddot{x} = J_A(q)K_p^{-1}J_A^T(q)h_A. \tag{7.13}
\]

An active compliance in the joint space has then been obtained. Notice that, in this case, the equivalent compliance matrix \( J_A(q)K_p^{-1}J_A^T(q) \) is always dependent on the manipulator configuration, both for the force and moment components. Also in this case, the occurrence of manipulator Jacobian singularities is to be analyzed apart.

### 7.3 Impedance Control

It is now desired to analyze the interaction of manipulator with environment under the action of an inverse dynamics control in the operational space. With reference to model (7.1), consider the control law (6.49)

\[
u = B(q)y + n(q, \dot{q}),
\]

with \( n \) as in (6.48). In the presence of end-effector forces, the controlled manipulator is described by

\[
\ddot{q} = y - B^{-1}(q)J^T(q)h
\]

that reveals the existence of a nonlinear coupling term due to contact forces. Choose \( y \) in a way conceptually analogous to (6.106) as

\[
y = J_A^{-1}(q)M_d^{-1}(M_d\ddot{x}_d + K_D\dot{x} + K_P\ddot{x}_d - M_dJ_A(q, \dot{q})\dot{\dot{q}}), \tag{7.15}
\]

where \( M_d \) is a positive definite diagonal matrix. Substituting (7.15) into (7.14) and accounting for second-order differential kinematics in the form (4.117) yields

\[
M_d\dddot{x}_d + K_D\ddot{x} + K_P\dot{x} = M_dB_A^{-1}(q)h_A, \tag{7.16}
\]

where

\[
B_A(q) = J_A^{-T}(q)B(q)J_A^{-1}(q)
\]

is the inertia matrix of the manipulator in the operational space as in (4.120); this matrix is configuration-dependent and is positive definite if \( J_A \) has full rank.

Eq. (7.16) establishes a relationship through a generalized mechanical impedance between the vector of resulting forces \( M_dB_A^{-1}h_A \) and the vector of displacements \( \ddot{x} \) in the operational space. This impedance can be attributed to a mechanical system characterized by a mass matrix \( M_d \), a damping matrix \( K_D \), and a stiffness matrix \( K_P \), which allow specifying the dynamic behavior along the operational space directions.

The presence of \( B_A^{-1} \) makes the system coupled. If it is wished to keep linearity and decoupling during interaction with the environment, it is then necessary to

the generalized contact force; this can be achieved by means of appropriate force sensors which are usually mounted on the manipulator wrist\(^1\). Choosing

$$u = B(q)y + n(q, \dot{q}) + J^T(q)h$$ (7.17)

with

$$y = J^{-1}_A(q)M_d^{-1}(M_d\ddot{x}_d + K_D\ddot{x} + K_P\ddot{x} - M_dJ_A(q, \dot{q})\dot{q} - h_A),$$ (7.18)

on the assumption of error-free force measurements, yields

$$M_d\dddot{x} + K_D\ddot{x} + K_P\ddot{x} = h_A.$$ (7.19)

It is worth noticing that the addition of the term \(J^T h\) in (7.17) exactly compensates the contact forces and then it renders the manipulator infinitely stiff with respect to external stress. In order to confer a compliant behavior to the manipulator, the term \(-J^{-1}_A M^{-1}_d h\) has been introduced in (7.18) which allows characterizing the manipulator as a linear impedance with regard to the equivalent forces \(h_A\), as shown in (7.19). The resulting block scheme of a manipulator in contact with an elastic environment under impedance control is illustrated in Fig. 7.2.

The behavior of system (7.19) at the equilibrium is analogous to that described by Eq. (7.2); nonetheless, compared to a compliance control specified by \(K_P\), Eq. (7.19) allows a complete characterization of system dynamics through an active impedance specified by matrices \(M_d, K_D,\) and \(K_P\). These matrices are usually taken as diagonal; also in this case, it is not difficult to recognize that impedance is configuration-independent as regards the force components, while it depends on the current manipulator configuration as regards the moment components through the matrix \(T_A\).

\(^1\) See the next chapter for a treatment of force sensors.
Furthermore, similarly to active and passive compliance, the concept of *passive impedance* can be introduced if the interaction force \( h_A \) is generated at the contact with an environment of proper mass, damping and stiffness. In this case, the system of manipulator with environment can be regarded as a mechanical system constituted by the parallel of the two impedances, and then its dynamic behavior is conditioned by the relative weight between them. As pointed out above, one may think about constructing a mechanical device with proper passive impedance that allows the manipulator to better cope with the given interaction task.

**Example 7.2.** Consider the planar arm in contact with an elastically compliant plane of the previous example. Apply the impedance control with force measurement (7.17) and (7.18) characterized by:

\[
M_d = \text{diag}\{m_{dx}, m_{dy}\} \quad K_D = \text{diag}\{k_{Dx}, k_{Dy}\} \quad K_P = \text{diag}\{k_{Pz}, k_{Py}\}.
\]

If \( x_d \) is constant, the dynamics of the manipulator and environment system along the two directions of the operational space is described by

\[
m_{dx} \ddot{x} + k_{Dx} \dot{x} + (k_{Pz} + k_z)x = k_z x_c + k_{Pz} x_d
\]

\[
m_{dy} \ddot{y} + k_{Dy} \dot{y} + k_{Py} y = k_{Py} y_d.
\]

Along the vertical direction, one has an unconstrained motion whose time behavior is determined by the following natural frequency and damping factor:

\[
\omega_{ny} = \sqrt{\frac{k_{Py}}{m_{dy}}} \quad \zeta_y = \frac{k_{Dy}}{2\sqrt{m_{dy}k_{Py}}}.
\]

while, along the horizontal direction, the behavior of the contact force \( f_x = k_z(x - x_c) \) is determined by

\[
\omega_{nx} = \sqrt{\frac{k_{Pz} + k_z}{m_{dx}}} \quad \zeta_x = \frac{k_{Dz}}{2\sqrt{m_{dx}(k_{Pz} + k_z)}}.
\]

Below, the dynamic behavior of the system is analyzed for two different values of environment compliance: \( k_z = 10^3 \text{ N/m} \) and \( k_z = 10^4 \text{ N/m} \). The actual arm is the same as in Example 4.2. Apply an impedance control with force measurements of the kind (7.17) and (7.18), and PD control actions equivalent to those chosen in the simulations of Section 6.7, i.e.,

\[
m_{dx} = m_{dy} = 100 \quad k_{Dx} = k_{Dy} = 500 \quad k_{Pz} = k_{Pz} = 2500.
\]

For these values it is

\[
\omega_{ny} = 5 \text{ rad/s} \quad \zeta_y = 0.5.
\]

Then, for the more compliant environment it is

\[
\omega_{nx} \approx 5.9 \text{ rad/s} \quad \zeta_x \approx 0.42,
\]
FIGURE 7.3
Time history of the tip position along vertical direction and of the contact force along horizontal direction with impedance control scheme for environments of different compliance.

whereas for the less compliant environment it is

\[ \omega_{nz} \approx 11.2 \text{ rad/s} \quad \zeta \approx 0.22. \]

Let the arm tip be in contact with the environment at position \( p = [1 \ 0]^T \); it is desired to take it to position \( p_d = [1.1 \ 0.1]^T \).

The results in Fig. 7.3 show that motion dynamics along the vertical direction is the same in the two cases. As regards the contact force along the horizontal direction, for the more compliant environment (dashed line) a well-damped behavior is obtained, whereas for the less compliant environment (solid line) the resulting behavior is less damped. Further, at the equilibrium, in the first case a displacement of 7.14 cm with a contact force of 71.4 N are observed, whereas in the second case a displacement of 2 cm with a contact force of 200 N are observed.

### 7.4 FORCE CONTROL

In the above schemes, the interaction force could be indirectly controlled by acting on the reference value \( x_d \) of the manipulator motion control system. Interaction between manipulator and environment is anyhow directly influenced by compliance of the environment and by either compliance or impedance of the manipulator.

If it is desired to accurately control the contact force, it is necessary to devise control schemes that allow directly specifying the desired interaction force. The development of a force control system, in analogy to a motion control system, would require the adoption of a stabilizing PD control action on the force error besides the usual nonlinear compensation actions. Force measurements are typically corrupted by noise, and then a derivative action cannot be implemented in practice. The stabilizing action is to be provided by suitable damping of velocity terms. As a consequence, a force control system typically features a control law based not only on force measurements but also on velocity measurements, and eventually position measurements, too.

The realization of a force control scheme can be entrusted to the closure of an outer force regulation feedback loop generating the control input for the motion control
scheme the manipulator is usually endowed with. Therefore, force control schemes are presented below which are based on the use of an inverse dynamics position control. Nevertheless, notice that a force control strategy is meaningful only for those directions of the operational space along which interaction forces between manipulator and environment may arise.

7.4.1 Force Control with Inner Position Loop

With reference to the inverse dynamics law with force measurement (7.17), choose in lieu of (7.18) the control

\[ y = J_A^{-1}(\dot{q})M_d^{-1}(-K_D\dot{x} + K_P(x_F - x) - M_d\ddot{J}_A(q, \dot{q}, \ddot{q})) \]  

(7.20)

where \( x_F \) is a suitable reference to be related to a force error. Notice that the control law (7.20) does not foresee the adoption of compensating actions relative to \( \dot{x}_F \) and \( \ddot{x}_F \). Substituting (7.20) into (7.17) leads, after similar algebraic manipulation as above, to the system described by

\[ M_d\ddot{x} + K_D\dot{x} + K_Px = K_Px_F, \]  

(7.21)

which shows how Eqs. (7.17) and (7.20) perform a position control taking \( x \) to \( x_F \) with a dynamics specified by the choice of matrices \( M_d, K_D, \) and \( K_P \).

Let \( h_{Ad} \) denote the desired constant force reference; the relation between \( x_F \) and the force error can be symbolically expressed as

\[ x_F = C_F(h_{Ad} - h_A), \]  

(7.22)

where \( C_F \) is a diagonal matrix whose elements give the control actions to perform along the operational space directions of interest. Eqs. (7.21) and (7.22) reveal that force control is developed on the basis of a preexisting position control loop.

On the assumption of the elastically compliant environment described by (7.7), Eq. (7.21) with (7.22) becomes

\[ M_d\ddot{x} + K_D\dot{x} + K_P(I + C_FK_A)x = K_PC_F(K_Ax_e + h_{Ad}). \]  

(7.23)
To decide about the kind of control action to specify with $C_F$, it is worth representing Eqs. (7.7), (7.21), and (7.22) in terms of the block scheme in Fig. 7.4, which is logically derived from the scheme in Fig. 7.2. This scheme suggests that if $C_F$ has a purely proportional control action, then $h_A$ cannot reach $h_{Ad}$, and $x_e$ influences the interaction force also at steady state.

If $C_F$ has also an integral control action on the components of generalized force, then it is possible to achieve $h_A = h_{Ad}$ at steady state and, at the same time, to reject the effect of $x_e$ on $h_A$. Hence, a convenient choice for $C_F$ is a proportional-integral (PI) action

$$C_F = K_F + K_I \int^t \delta(\cdot) d\xi.$$  

The dynamic system resulting from (7.23) and (7.24) is of third order, and then it is necessary to adequately choose the matrices $K_D$, $K_P$, $K_F$, and $K_I$ in respect of the characteristics of the environment. Since the values of environment stiffness are typically high, the weight of the proportional and integral actions shall be contained; the choice of $K_F$ and $K_I$ influences the stability margins and the bandwidth of the system under force control. On the assumption that a stable equilibrium is reached, it is $h_{A\infty} = h_{Ad}$ and then

$$K_A x_{\infty} = K_A x_e + h_{Ad}.$$  

### 7.4.2 Force Control with Inner Velocity Loop

From the block scheme of Fig. 7.4 it can be observed that, if the position feedback loop is opened, $x_F$ represents a velocity reference, and then an integration relationship exists between $x_F$ and $x$. This leads to recognizing that, in this case, the interaction force with the environment coincides with the desired value at steady state, even with a proportional force controller $C_F$. In fact, choosing

$$y = J_A^{-1}(q)M_d^{-1}(-K_D \dot{x} + K_P x_F - M_d \dot{J}_A(q, \dot{q})q)$$  

with a purely proportional control structure ($C_F = K_F$) on the force error yields

$$x_F = K_F(h_{Ad} - h_A).$$
FIGURE 7.6
Block scheme of parallel force/position control.

and then system dynamics is described by

\[ M_d \ddot{x} + K_D \dot{x} + K_P K_F K_A x = K_P K_F (K_A x_e + h_{Ad}). \] (7.28)

The relationship between position and contact force at the equilibrium is given by (7.25). The corresponding block scheme is reported in Fig. 7.5. It is worth emphasizing that control design is simplified, since the resulting system now is of second order\(^2\).

7.4.3 Parallel Force/Position Control

The presented force control schemes require the force reference to be consistent with the geometrical features of the environment. In fact, if \( h_{Ad} \) has components outside \( \mathcal{R}(K_A) \), both Eq. (7.23) (in case of an integral action in \( C_F \)) and Eq. (7.28) show that, along the corresponding operational space directions, the components of \( h_{Ad} \) are interpreted as velocity references which cause a drift of the end-effector position. If \( h_{Ad} \) is correctly planned along the directions outside \( \mathcal{R}(K_A) \), the resulting motion governed by the motion control action tends to take the end-effector position to zero in the case of (7.23), and the end-effector velocity to zero in the case of (7.28). Hence, the above control schemes do not allow motion control even along the admissible task space directions.

If it is desired to specify a desired end-effector location \( x_d \) as in pure motion control schemes, the scheme of Fig. 7.4 can be modified by adding the reference \( x_d \) to the input where positions are summed. This corresponds to choosing

\[ y = J^{-1}_A(q) M^{-1}_d (-K_D \dot{x} + K_P (\ddot{x} + \dot{x}_F) - M_d \dot{J}_A(q, \dot{q}) \ddot{q}) \] (7.29)

where \( \ddot{x} = x_d - x \). The resulting scheme (Fig. 7.6) is termed parallel force/position control, in view of the presence of a position control action \( K_F \ddot{x} \) in parallel to a force

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\(^2\) In reality, the matrices \( K_P \) and \( K_F \) are not independent and one may refer to a single matrix \( K'_F = K_P K_F \).
control action $K_FC_F(h_{Ad} - h_A)$. It is easy to verify that, in this case, the equilibrium position satisfies the equation

$$\mathbf{x}_\infty = \mathbf{x}_d + C_F (K_A (\mathbf{x}_e - \mathbf{x}_\infty) + h_{Ad}).$$  \hspace{1cm} (7.30)

Therefore, along those directions outside $R(K_A)$ where motion is unconstrained, the position reference $\mathbf{x}_d$ is reached by $\mathbf{x}$. Vice versa, along those directions in $R(K_A)$ where motion is constrained, $\mathbf{x}_d$ is treated as an additional disturbance; the adoption of an integral action in $C_F$ as for the scheme of Fig. 7.4 ensures that the force reference $h_{Ad}$ is reached at steady state, at the expense of a position error on $\mathbf{x}$ depending on environment compliance.

**Example 7.3.** Consider again the planar arm in contact with the elastically compliant plane of the above examples; let the initial contact position be the same as that of Example 7.2. Performance of the various force control schemes is analyzed; as in Example 7.2, a more compliant ($k_e = 10^3$ N/m) and a less compliant ($k_e = 10^4$ N/m) environment are considered. The position control actions $M_d$, $K_D$, $K_F$ are chosen as in Example 7.2; a force control action is added along the horizontal direction, i.e.,

$$C_F = \text{diag}(c_{Fx}, 0).$$

The reference for the contact force is chosen as $h_{Ad} = [10 \ 0]^T$; the position reference—meaningful only for the parallel control—is taken as $p_d = [1.015 \ 0.1]^T$.

With regard to the scheme with inner position loop of Fig. 7.4, a PI control action $c_{Fx}$ is chosen with parameters:

$$k_{Fx} = 0.00064 \quad k_{Ix} = 0.0016.$$  

This confers two complex poles $(-1.96, \pm j5.74)$, a real pole $(-1.09)$, and a real zero $(-2.5)$ to the overall system, for the more compliant environment.

With regard to the scheme with inner velocity loop of Fig. 7.5, the proportional control action is

$$k_{Fv} = 0.0024$$

so that the overall system, for the more compliant environment, has two complex poles $(-2.5, \pm j7.34)$. 

**FIGURE 7.7**

Time history of the tip position and of the contact force along horizontal direction with force control scheme with inner position loop for two environments of different compliance.
FIGURE 7.8
Time history of the tip position and of the contact force along horizontal direction with force control scheme with inner velocity loop for two environments of different compliance.

FIGURE 7.9
Time history of tip position and of the contact force along horizontal direction with parallel force/position control scheme for two environments of different compliance.

With regard to the parallel control scheme of Fig. 7.6, the PI control action $c_{F,a}$ is chosen with the same parameters as for the first control scheme.

Figs. 7.7, 7.8, and 7.9 report the time history of the tip position and contact force along axis $x$ for the three considered schemes. A comparison between the various cases shows what follows:

- All control laws guarantee a steady-state value of contact forces equal to the desired one for both the more compliant (dashed line) and the less compliant (continuous line) environment.
- For given motion control actions $(M_d, K_D, K_P)$, the force control with inner velocity loop presents a faster dynamics than that of the force control with inner position loop.
- The dynamic response with the parallel control shows how the addition of a position reference along the horizontal direction degrades the transient behavior, but it does not influence the steady-state contact force. This effect can be justified by noting that a step position input is equivalent to a properly filtered impulse force input.
The reference position along axis $y$ is obviously reached by the arm tip according to dynamics of position control; the relative time history is not reported.

7.5 NATURAL CONSTRAINTS AND ARTIFICIAL CONSTRAINTS

Interaction control schemes can be employed for execution of constrained motions as long as the force and position references are chosen to be compatible with environment geometry.

A real manipulation task is characterized by complex contact situations where some directions are subject to end-effector position constraints and others are subject to interaction force constraints. During task execution, the nature of constraints may vary substantially.

The need to handle complex contact situations requires the capability to specify and perform control of both end-effector position and contact force. An example is that of a surface finishing task where the tool motion is specified in the direction tangent to the piece, while along the normal direction it is desired to exert a force of given value.

A fundamental aspect to be considered is that it is not possible to simultaneously impose arbitrary values of position and force along each direction. As a consequence, one should ensure that the reference trajectories for the control system be compatible with the constraints imposed by the environment during task execution, so as to achieve a correct specification of the control problem.

A kineto-statics analysis of a situation of interaction between the manipulator and environment leads to the following considerations.

- Along each degree of freedom of the task space, the environment imposes either a position or a force constraint to the manipulator’s end effector; such constraints are termed natural constraints since they are determined directly by task geometry.

- Along each degree of freedom of the task space, the manipulator can control only the variables which are not subject to natural constraints; the reference values for those variables are termed artificial constraints since they are imposed with regard to the strategy for executing the given task.

Notice that the two sets of constraints are complementary, in that they regard different variables for each degree of freedom. Also they allow a complete specification of the task, since they involve all variables. Kineto-statics variables are associated with these constraints, i.e., velocities and generalized forces.

It should be pointed out that the above examples illustrating performance of motion and force control strategies have implicitly accounted for the presence of natural and artificial constraints. In view of the particular simple task geometry, the degrees of freedom were naturally described with reference to the base frame assumed to express operational space quantities.

In the general case, it is worth introducing a constraint frame $O_c=x_c y_c z_c$, not necessarily aligned with the base frame, in order to simplify task description and to
allow determination of natural constraints and then specification of artificial constraints. The introduction of the constraint frame noticeably simplifies task planning, but the control strategy will obviously have to account for the rotation matrix—eventually time-varying during task execution—which is needed to transform quantities expressed in the base frame into the corresponding quantities in the constraint frame.

### 7.5.1 Case Studies

To illustrate description of an interaction task in terms of natural and artificial constraints as well as to emphasize the opportunity to use a constraint frame, in the following a number of typical case studies are analyzed.

**Sliding on a Planar Surface.** The end-effector manipulation task is the sliding of a prismatic object on a planar surface. Task geometry suggests choosing the constraint frame as attached to the contact plane with an axis orthogonal to the plane (Fig. 7.10).

Natural constraints can be determined first. Motion constraints describe the impossibility to generate arbitrary linear velocity along axis $z_c$ and angular velocities about axes $x_c$ and $y_c$; if the plane is rigid, then these velocities are null. Force constraints describe the impossibility to exert arbitrary forces along axes $x_c$ and $y_c$ and moment about axis $z_c$; if the plane is frictionless, these generalized forces are zero.

The artificial constraints regard the variables not subject to natural constraints. Hence, with reference to the natural generalized force constraints along axes $x_c$, $y_c$ and about $z_c$, it is possible to specify artificial constraints for linear velocity along $x_c$, $y_c$ and angular velocity about $z_c$. Similarly, with reference to natural velocity constraints along axis $z_c$ and about axes $x_c$, $y_c$, it is possible to specify artificial constraints for force along $z_c$ and moments about $x_c$, $y_c$. The set of constraints is summarized in the table in Fig. 7.10.

**Peg-in-Hole.** The end-effector manipulation task is the insertion of a cylindrical object (peg) in a hole. Task geometry suggests choosing the constraint frame with an axis parallel to the hole axis and the origin along this axis (Fig. 7.11).
The natural constraints are determined by observing that it is not possible to generate arbitrary linear velocities along axes \( x_c, y_c \) and angular velocity about the same axes, nor is possible to exert arbitrary force along \( z_c \) and moment about \( z_c \); all these variables are null in the case of rigid frictionless insertion. As a consequence, the artificial constraints allow specifying forces along \( x_c, y_c \) and moments about the same axes, as well as linear velocity along \( z_c \) and angular velocity about the same axis. The table in Fig. 7.11 summarizes the constraints. Among the variables subject to artificial constraints, \( \dot{p}_z^c \neq 0 \) describes insertion while the others are typically null to effectively perform the task.

**Turning a Crank.** The end-effector manipulation task is the turning of a crank. Task geometry suggests choosing the constraint frame with an axis aligned with the axis of the idle handle and another axis aligned along the crank lever (Fig. 7.12). Notice that in this case the constraint frame is time-varying.

The natural constraints do not allow generating arbitrary linear velocities along \( x_c, z_c \) and angular velocities about \( x_c, y_c \), nor arbitrary force along \( y_c \) and moment about \( z_c \). As a consequence, the artificial constraints allow specifying forces along \( x_c, z_c \) and moments about \( x_c, y_c \), as well as a linear velocity along \( y_c \) and an angular velocity about \( z_c \). The situation is summarized in the table in Fig. 7.12. Among the variables subject to artificial constraints, forces and moments are typically null for task execution.

### 7.6 HYBRID FORCE/POSITION CONTROL

Description of an interaction task between manipulator and environment in terms of natural constraints and artificial constraints, expressed with reference to the constraint frame, suggests a control structure that utilizes the artificial constraints to specify the objectives of the control system and allows controlling only those variables not subject to natural constraints. In fact, the control action shall not affect those variables
constrained by the environment so as to avoid conflicts between control and interaction with environment that may lead to an improper system behavior. Such a control structure is termed hybrid force/position control, since definition of artificial constraints involves both force and position variables.

The natural constraints are described by specifying a set of components of velocity and generalized force vectors expressed in the constraint frame. Such constraints can be written in compact form as

$$\Sigma v^c = v_n^c \quad (I - \Sigma) h^c = h_n^c,$$  \hspace{1cm} (7.31)$$

where $\Sigma$ is a diagonal matrix with either unit or null elements corresponding to the task components to be constrained. In (7.31), the natural constraints imposed by the environment are specified by $v_n^c$ and $h_n^c$; it is easy to find the matrix $\Sigma$ for the case studies presented above. Matrices $\Sigma$ and $(I - \Sigma)$ are complementary, since for each degree of freedom one has a natural constraint regarding either a velocity or a force component.

Once the natural constraints have been expressed in the above form, the set of artificial constraints can be expressed in the form

$$(I - \Sigma)v^c = v_n^c \quad \Sigma h^c = h_n^c,$$  \hspace{1cm} (7.32)$$

where $v_n^c$ and $h_n^c$ represent the components of velocity and generalized force that are taken to describe the artificial constraints. The matrix $\Sigma$ is called selection matrix, since it allows selecting the appropriate control actions for each degree of freedom of the task.

The general block scheme of a hybrid force/position control structure is shown in Fig. 7.13. It is assumed that all the output variables needed for force/position control are available.

Regarding motion control actions, in general, the end-effector position error $\bar{x}$ and velocity error $\bar{v}$ are to be fed back; velocity and acceleration references are available at the input, once the artificial constraints have been specified. Regarding interaction
control actions, in general the end-effector position error $\bar{x}$ and velocity $\dot{x}$ are to be fed back besides the contact force $h_A$. The force reference is available at the input, once the artificial constraints have been specified.

At first, it is necessary to use the matrix $T_A$ to transform $\bar{x}$ and $\dot{x}$ into quantities homogeneous to those of the artificial constraints. To this regard, for limited position errors, it is reasonable to assimilate $\bar{x}$ to a velocity vector and treat it in the same fashion as $\dot{x}$. The transformation formally operates also on the force $h_A$, although in reality the physical force $h$ is available directly\(^3\).

The obtained quantities are expressed with reference to the manipulator base frame; hence, it is necessary to transform them in the constraint frame where the task is naturally described. This is achieved by premultiplying them by the rotation matrix $R^c$ transforming a vector from the base frame to the constraint frame. This transformation allows guaranteeing an effective decoupling of the task degrees of freedom through the use of selection matrices.

On the basis of the selection operated by the matrices $\Sigma$ and $(I - \Sigma)$ on the feedback quantities as well as on the corresponding references in view of the artificial constraints, it is possible to design interaction control and motion control actions along complementary directions of the task space. The actual control schemes to employ can be chosen among the variety of schemes presented in the above sections and in the previous chapter, with the notable exception of the parallel control which is to be regarded as an alternative solution to hybrid control.

The output vectors $y^c_F$ and $y^c_F$ of the control blocks can be summed to generate

\(^3\) As will be seen in the next chapter, force measurements are typically available with reference to a frame attached to the force sensor mounted on the manipulator's wrist.
the control $y^c$ expressing the actions on all task components. This vector is then to be transformed from the constraint frame back to the base frame by premultiplying its linear and angular components by the matrix $R_e = (R^c)^T$.

Finally, depending on the actual control scheme, it may be necessary to perform (partial or total) nonlinear compensating actions for both force and position. The resulting control law gives the joint control torques at the input of the manipulator.

According to the above analysis, the control structure is adapted to the task requirements, thanks to the use of selection matrices. Along each task component, the proper control action is activated by the selection matrix, while its dual one is ignored; this strategy avoids undesirable interference between the two controllers, since it structurally decouples force control actions from motion control actions in terms of the components of the given task.

The above considerations are founded on the assumption of perfect task planning. In those situations when hybrid control has to operate under imperfect task planning, the system behavior may become quite critical. For instance, consider the extremal case when a hybrid controller governs manipulator motion in a situation of unplanned impact. As the selection matrix mechanism performs a model-based control logic which is not supported by any verification during task execution, it is not possible to modify the behavior of the control scheme in respect of what actually happens in the environment. In fact, the selection matrices cancel part of the force sensor measurements on the assumption that this information is not useful to the controller, whereas the same information might be crucial in all such circumstances when lack of information about the environment plays a significant role.

An effective alternative to the hybrid control strategy in situations of a poorly structured environment can be provided by the parallel control strategy presented in Section 7.4.3. The absence of selection mechanisms on the force and position feedback loops confers to the parallel control a capacity of autonomous reaction whenever the natural constraints are violated, that is, when there is mismatching between planning and real geometry of the interaction.

**Example 7.4.** Consider a two-link planar arm in contact with a purely frictionless elastic plane; differently from the above examples, the plane is at an angle of $\pi/4$ with axis $x$ (Fig. 7.14). The natural choice of the constraint frame is that with axis $x_c$ along the plane and axis $y_c$ orthogonal to the plane; the task is obviously characterized by two degrees of freedom. Let $p^c$ and $f^c$ respectively denote the tip position and contact force, both expressed in the constraint frame; then, the selection matrix is

$$
\Sigma = \text{diag}(0, 1),
$$

since the natural constraints regard $y^c$ and $f^c_x$. As a consequence, the artificial constraints allow specifying $f^c_y$ and $\dot{x}^c$. If the task is to slide the tip along the plane, the constraint frame orientation remains constant with respect to the base frame. The relative rotation matrix is given by

$$
R_e = \begin{bmatrix}
1/\sqrt{2} & 1/\sqrt{2} \\
-1/\sqrt{2} & 1/\sqrt{2}
\end{bmatrix}.
$$

(7.33)

According to the hybrid control structure, it is possible to perform motion control along axis $x_c$ and interaction (compliance, impedance or force) control along axis $y_c$. Hence,
once the measured quantities expressed in the base frame—position error \( \hat{z} \), velocity \( \hat{x} \), and contact force \( f_y \)—are transformed into the constraint frame, it is possible to proceed as in the above examples. The resulting control law will have components along \( x_c, y_c \), and then it will have to be transformed back to the base frame via the matrix \( R_c \).

PROBLEMS

7.1. Show that the equilibrium position and force for the compliance control scheme are expressed by (7.9) and (7.10), respectively.

7.2. Show that the equilibrium position for the parallel force/position control scheme satisfies (7.30).

7.3. For the manipulation task of driving a screw in a hole illustrated in Fig. 7.15, find the natural constraints and artificial constraints with respect to a suitably chosen constraint frame.

7.4. Consider the planar arm in contact with the elastically compliant plane in Fig. 7.14. The plane forms an angle of \( \pi/4 \) with axis \( x \) and its undeformed position intersects axis \( z \) in the point of coordinates \( (1, 0) \); environment stiffness along axis \( y_e \) is \( 5 \times 10^3 \) N/m. With the data of the arm in Section 6.7, design a hybrid control in which an inverse dynamics position control operates along axis \( x_c \) while an impedance control operates along axis \( y_c \). Perform a computer simulation of the interaction of the controlled manipulator along the rectilinear path from position \( p_i = [1 + 0.1\sqrt{2}, 0]^T \) m to \( p_f = [1.2 + 0.1\sqrt{2}, 0.2]^T \) m with a trapezoidal velocity profile and a trajectory duration \( t_f = 1 \) s. Implement the control in discrete-time with a sampling time of 1 ms.

7.5. For the arm and environment of Problem 7.4, design a hybrid control in which an inverse dynamics position control operates along axis \( x_c \), while a force control with inner position loop operates along axis \( y_c \); let the desired contact force along axis \( y_c \) be 50 N. Perform a computer simulation of the interaction of the controlled manipulator along the trajectory of Problem 7.4. Implement the control in discrete-time with a sampling time of 1 ms.

7.6. For the arm and environment of Problem 7.4, design a hybrid control in which an inverse dynamics position control operates along axis \( x_c \), while a force control with inner velocity loop operates along axis \( y_c \); the force reference is the same as in Problem 7.5. Perform a computer simulation of the interaction of the controlled manipulator along the trajectory of Problem 7.4. Implement the control in discrete-time with a sampling time of 1 ms.
FIGURE 7.15
Driving a screw in a hole.

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